Modeling Techniques
Models of Corporations

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Modeling Techniques through Models of Corporate Taxation

Session 1

0 Introduction—the power of simple.

A Incidence

1 Supply and demand (Berger and Seegert, 2023)

2 Who pays the tax?

3 Can Monopolists push the tax onto consumers?

4 Extensions: salience, overshifting, evasion, and empirical considerations(Bradley and Feldman, 2020; Kopczuk, Marion, Muehlegger, and Slemrod, 2013; Mace, Patel, and Seegert, 2020)

B Foundation of corporate models

1 Foundation of corporate models, Fisher's separation theorem (Fisher, 1930)

2 Two-period model (Modigliani and Miller, 1958)

3 Corporate taxes

Introduction—the power of simple

Models are simplifications of the world

“What a useful thing a pocket-map is!" I remarked.

“That’s another thing we’ve learned from your Nation," said Mein Herr, “map-making. But we’ve carried it much further than you. What do *you* consider the *largest* map that would be really useful?"

“About six inches to the mile."

“Only *six* inches!" exclaimed Mein Herr. “We very soon got to six yards to the mile. Then we tried a *hundred* yards to the mile. And then came the grandest idea of all! We actually made a map of the country, on the scale of a *mile to the mile*!"

“Have you used it much?" I inquired.

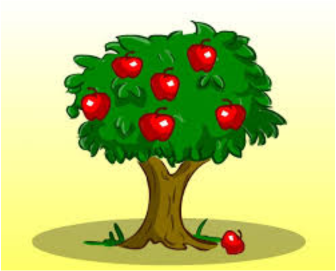
“It has never been spread out, yet," said Mein Herr: "the farmers objected: they said it would cover the whole country, and shut out the sunlight! So we now use the country itself, as its own map, and I assure you it does nearly as well."

from Lewis Carroll, Sylvie and Bruno Concluded, Chapter XI, London 1895

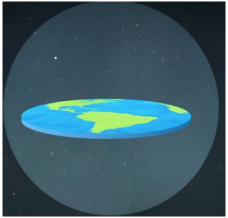
What explains why apples fall from their tree?

Different disciplines, different models

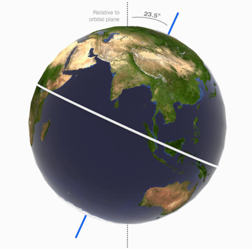
1. Physics: gravity.
2. Evolutionary biologist: trees that shot their apples upward into space did not propagate.
3. Economist: trees just responded to positive incentives to drop fruit to earth.
4. Accounting?



Consider the distance between the Univ. Utah and BYU



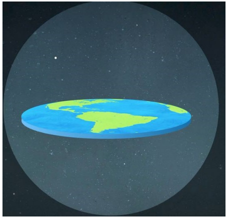
1. Flat earth model.
   - 36.41 miles



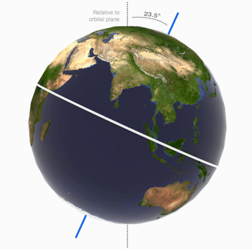
2. Spherical earth model.
   - 36.44 miles

- If you do not plan on going very far, flat earth is a fine model.

Consider the distance between the Univ. Utah and Tokyo



1. Flat earth model.
   - 5110.88 miles miles



2. Spherical earth model.
   - 5479.873 miles

- In far distances may need a better model.

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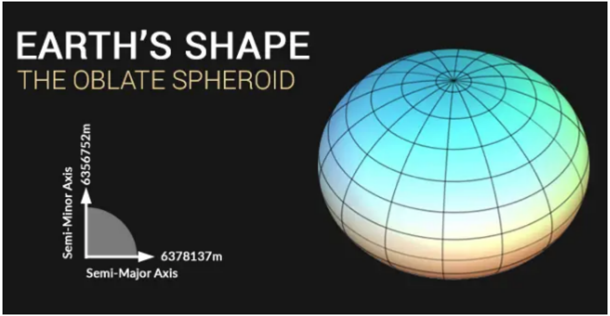
Consider the distance between the Univ. Utah and Tokyo



1. Flat earth model.
    - 5110.88 miles miles
2. Spherical earth model.
    - 5479.873 miles
3. Vincenty oblate spheroid earth model
    - 5492.64 miles

- How detailed of a model do you need?

Which of the following best describes you

- I have never written a model before
- I sometimes use models in my research
- I always model my research question
- I misunderstood what this modeling workshop was about

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Preliminary building blocks

Models need three things

1. Players—who is making a decision (e.g., firm, shareholder, CEO).
2. Strategies—what can the players do (e.g., choose investment levels).
3. Payoffs—what do the players receive (e.g., firm value or utility).

In my writing, I like to spell these out right away and in this order.

Preliminary building blocks

Models are used to highlight trade offs

1. Is your model about a new trade off? (e.g., dividends versus mergers).
2. Is your model about a new feature that affects the tradeoff (e.g., information revelation).
3. Make sure everything supports the novel aspect of your model.

Models start out simple and progress as we add features

1. We will start with the very basic models.

2. These models will be missing a lot of important details.

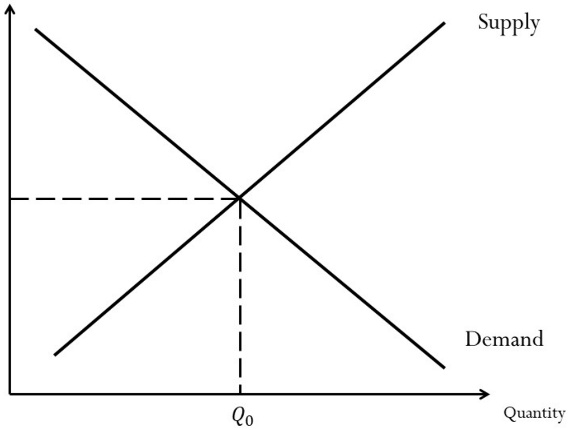
3. The hope is that these models can be the jumping off point for you to use in your own work

4. and the tools we learn can help build hypotheses from these models.

A.1 Supply and demand

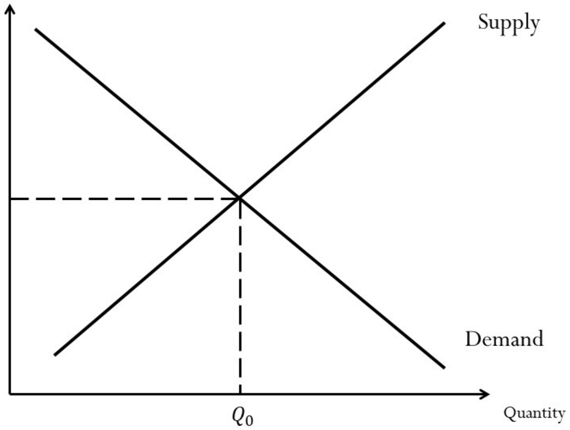
Berger and Seegert (2023)

Start with our supply and demand model



1. Players: consumers and producers.
2. Strategies: Buy or produce.
3. Payoffs: surplus from buying or producing.

Market equilibrium sets supply equal to demand



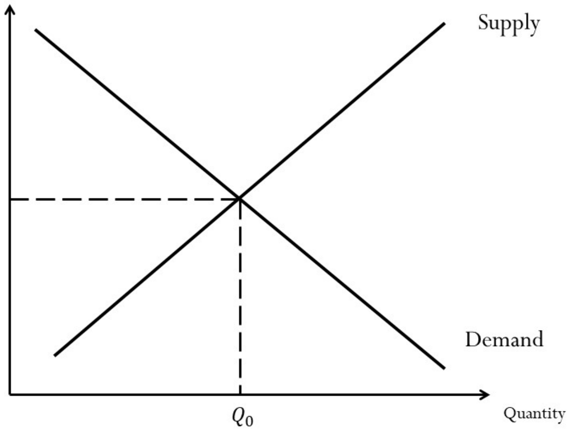
1. Supply curve p = q.
2. Inverse demand curve p = 100 - q
3. Market sets demand = supply

$$q = 100 - q$$

$$q = 50$$

$$p = 50$$

Now add taxes to the model



1. Supply curve p = q.
2. Inverse demand curve p = 100 - q
3. Now, add in tax and see how that changes behavior and payoffs.

How do you add taxes into this model?

- Shift the supply curve
- Shift the demand curve
- Add a wedge between supply and demand
- Add a new line representing the government's budget

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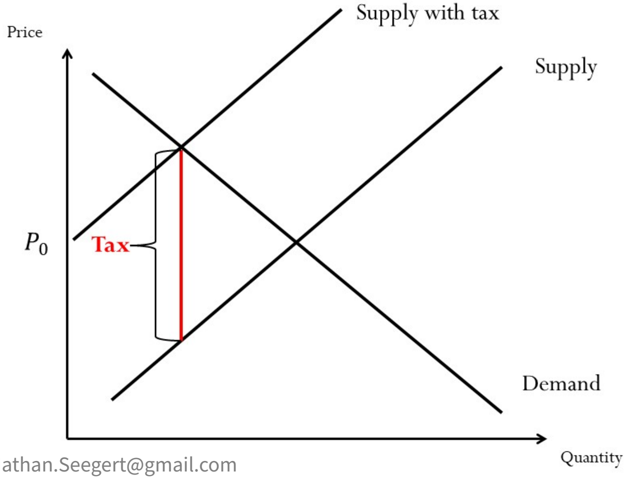
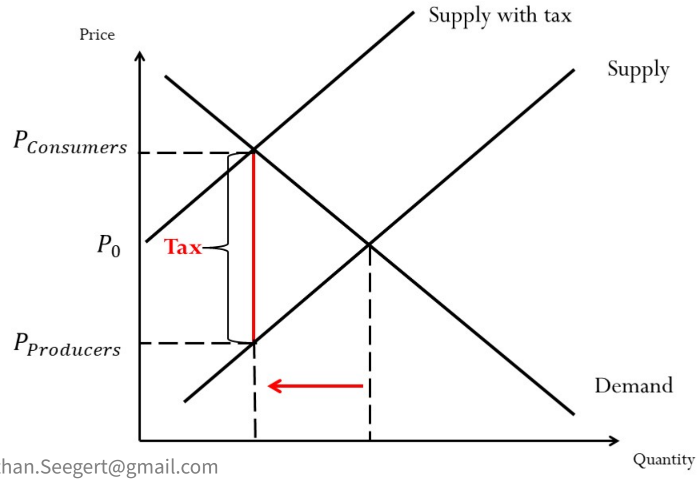
Taxes are a shift in the marginal cost



1. Taxes increase the marginal cost.
2. Marginal cost increases are a positive shift of the supply curve.
3. Supply curve p = q + t.

Incidence is the change in prices



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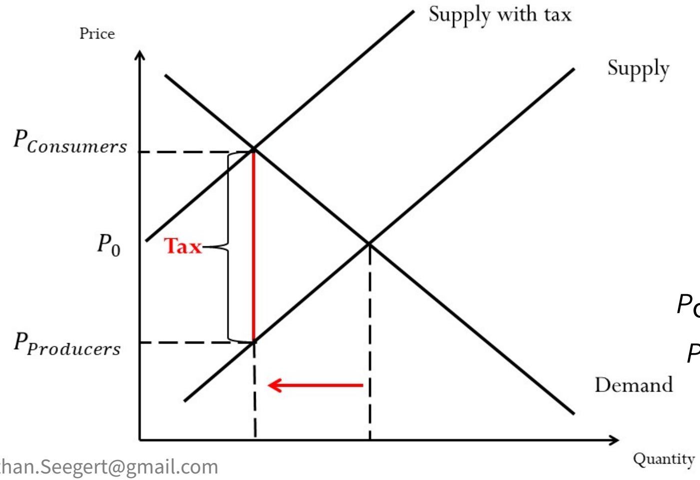
Here, taxes are evenly split



$$\text{supply} = \text{demand}$$

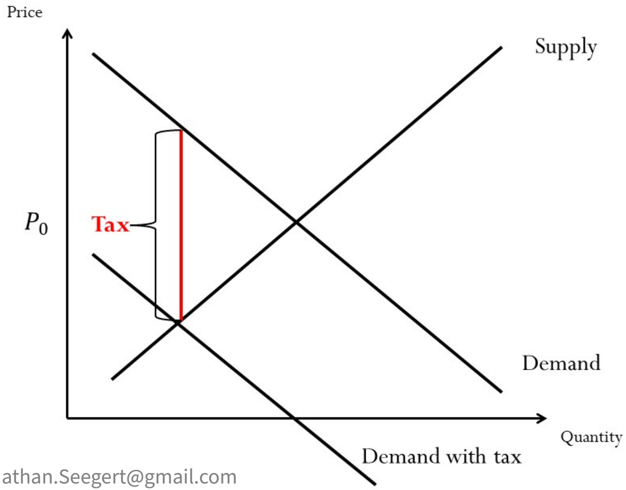
$$q + t = 100 - q$$

$$q = 50 - t/2$$

$$P_{\text{consumers}} = 100 - (50 - t/2)$$

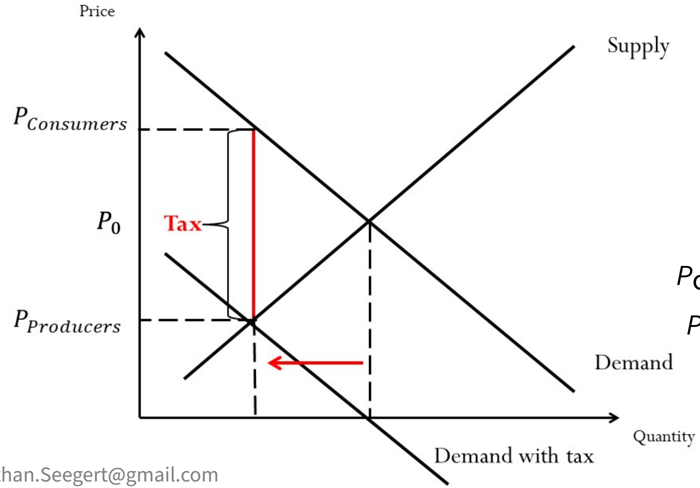
$$P_{\text{producers}} = (50 - t/2)$$

Wait, taxes decrease demand



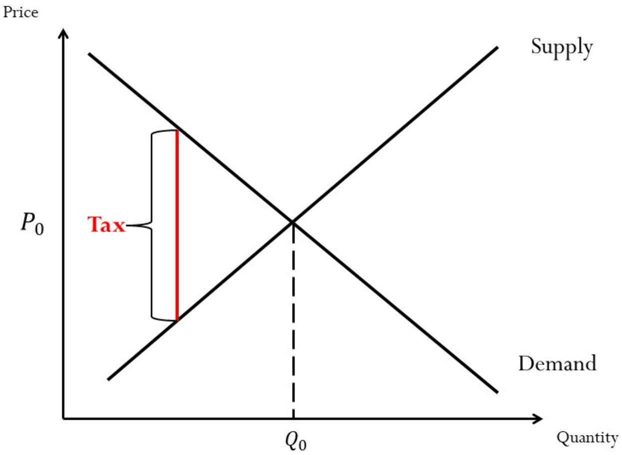
1. Taxes decrease demand.
2. Inverse demand curve p = 100 - q - t

Who pays the tax when taxes decrease demand?



$$\text{supply} = \text{demand}$$
$$q = 100 - q - t$$
$$q = 50 - t/2$$
$$P_{\text{consumers}} = 100 - (50 - t/2)$$
$$P_{\text{producers}} = (50 - t/2)$$

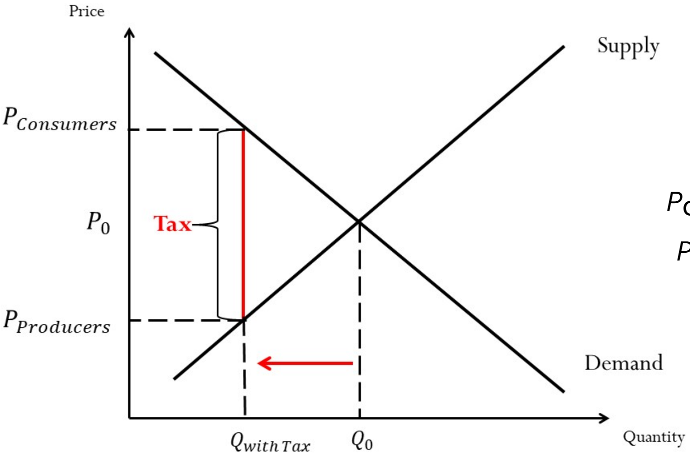
Wait, taxes create a wedge between supply and demand



$$\text{supply} = \text{demand}$$
$$q = 100 - q - t$$
$$q + t = 100 - q$$

You can add the tax on the RHS or the LHS—but not both

In these cases, consumers and producers perfectly split tax



$$P_{\text{consumers}} = 100 - (50 - t/2)$$

$$P_{\text{producers}} = (50 - t/2)$$

All of these ways produced the same model

Note, you can model taxes in any of those three ways

1. Shift left in supply.
2. Shift left in demand.
3. Wedge between supply and demand.

Sometimes there are multiple ways of modeling something.

It is important to know how sensitive the model is to these choices.

1. Cournot built a model of competition where firms competed over quantity.

2. Bertrand criticized Cournot suggesting firms should compete over prices.

In the case of competition, the strategy firms compete over have very different implications.

What if we are interested in welfare effects—not just prices?

1. In general, the change in price we focused on will give a different split than welfare changes.

2. The advantage of price changes is that we make very few assumptions.

3. The advantage of welfare changes is that welfare is ultimately what matters.

Changes in welfare calculations rely on assumptions to provide context for your paper.

The effects of regulation Berger and Seegert (2023)

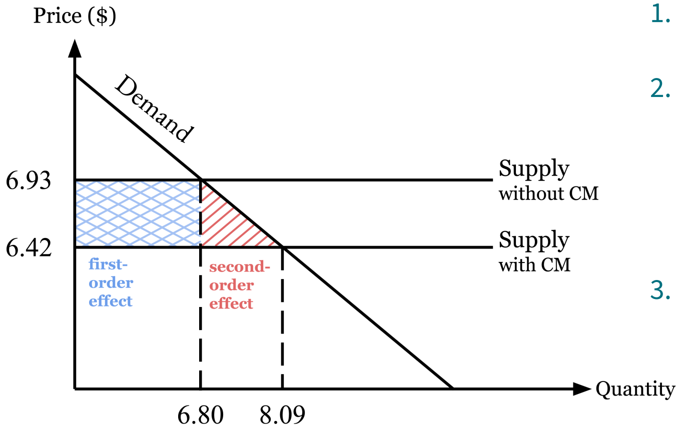
Marijuana industry some firms have cash management from banks and some do not.

1. Players: retail firms and wholesalers.
2. Strategies: how much to produce (wholesalers) and buy (retailers).
3. Payoffs: welfare from market transactions.

We are interested in retail firm behavior.

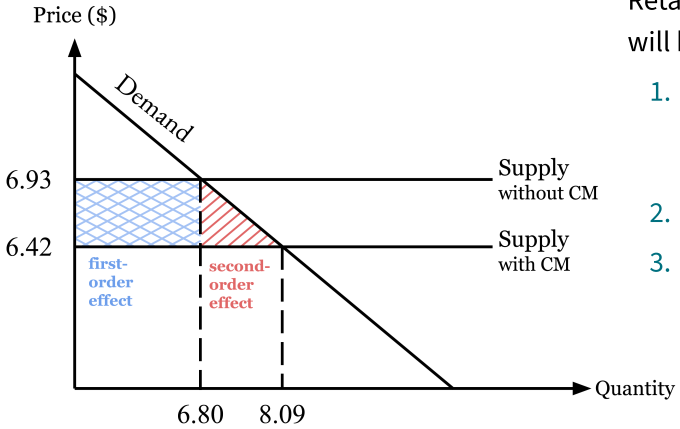
- Assume supply from wholesalers is perfectly elastic.
- Assume that without cash management marginal costs are higher.

Wholesale market supply and demand



1. Standard demand curve.
2. Perfectly elastic supply.
   - Simplifying assumption, good/bad?
   - How does it change the analysis?
3. Lack of cash management modeled as an additional marginal cost (tax)
   - Shift up in the supply curve.

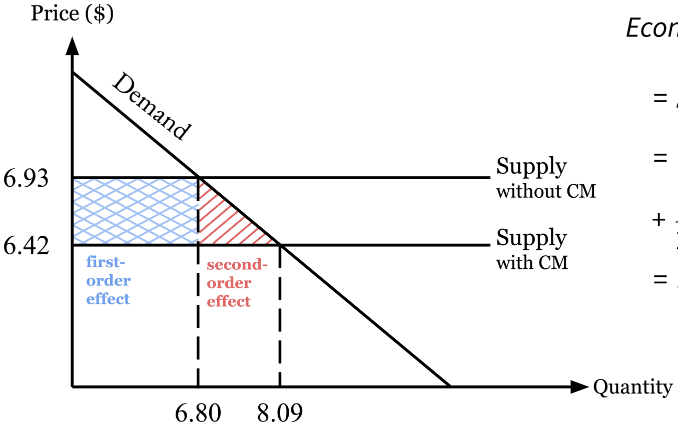
Predictions from the wholesale market



Retail firms with cash management will have

1. Lower wholesale prices for the same product.
2. Buy more product.
3. Lack of cash management leads to a cost of higher wholesale prices and fewer products bought.

Estimating costs from the wholesale market



Economic value wholesale market

$$= \Delta P_w \times q_w(\theta_w > 0) + \frac{1}{2}\Delta P_w \Delta q_w$$

$$= (6.93 - 6.42) \times 6.8$$

$$+ \frac{1}{2}(6.93 - 6.42) \times (8.09 - 6.80)$$

$$= \$3.80$$

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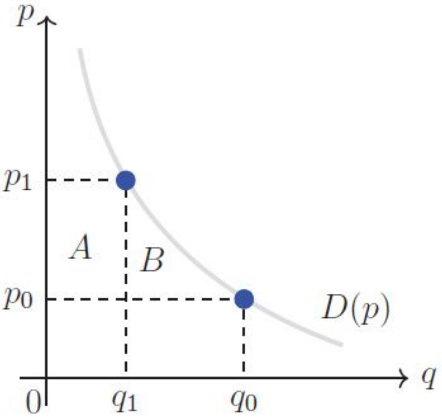
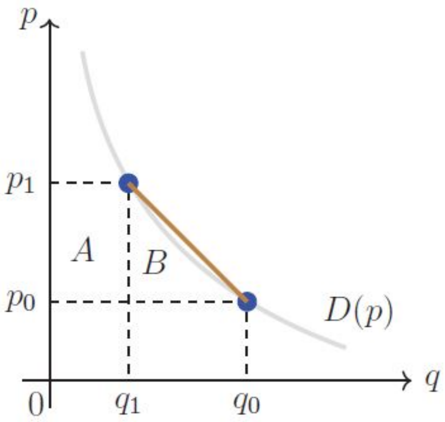
Relax linearity assumption



Kang and Vasserman (2022)

1. Bounds on the change in welfare.
2. Tighter bounds with different assumptions.

Relax linearity assumption

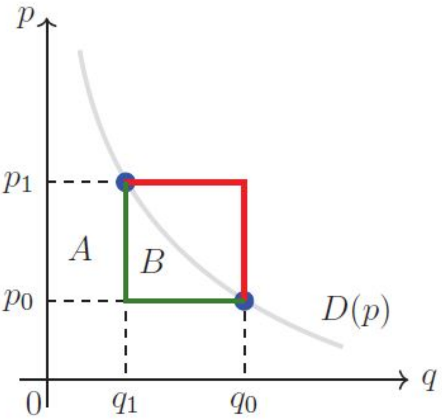


Kang and Vasserman (2022)

1. Linear assumption is wrong, but maybe close?

2. Note, it does not affect the first-order effect in A.

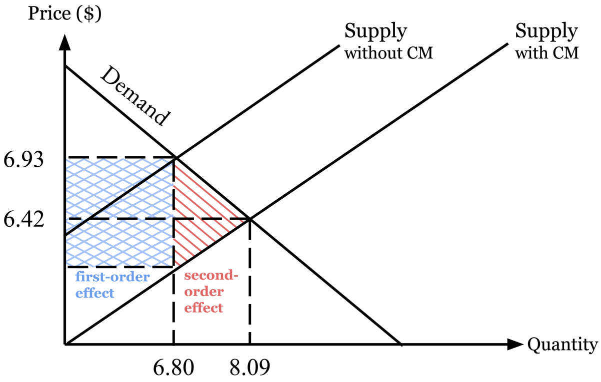
Relax linearity assumption



Kang and Vasserman (2022)

1. Widest bounds would be given by red (double linear) and green (zero).
2. Tighter bounds with different assumptions.

Relax perfectly elastic supply



1. Same predictions on price and quantity.
2. Costs from lack of cash management larger.
3. Our assumption provides a conservative lower bound.

The value of cash management

Berger and Seegert (2023) finds that the value of cash management in the marijuana industry in Washington is substantial,

- Total value $18,000,000 or 1.8% of total industry sales.
- $6,000,000 in the wholesale market
- $12,000,000 in the retail market.

Usefulness of a model increases with its simplicity

1. What is the simplest your model can be to show the result you want to highlight?

2. Is your model robust to more realistic assumptions?

Remember the goal of the model is to *clarify* the tradeoff you are studying.

A.2 Who pays the tax

In which market will consumers pay more of the tax?

Gasoline market

Hotel market

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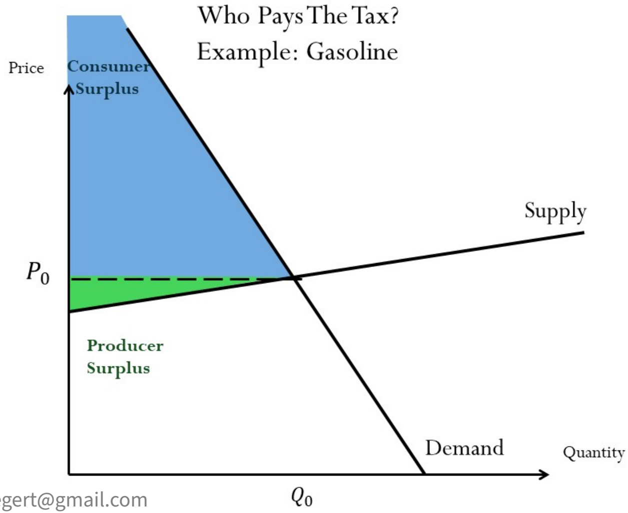
Change the model

How can you change the model to get consumers or producers to pay more of the tax?

1. Try the model with more *inelastic demand.*
   - e.g., gasoline market.

2. Try the model with more *inelastic supply.*
   - e.g., hotel market.

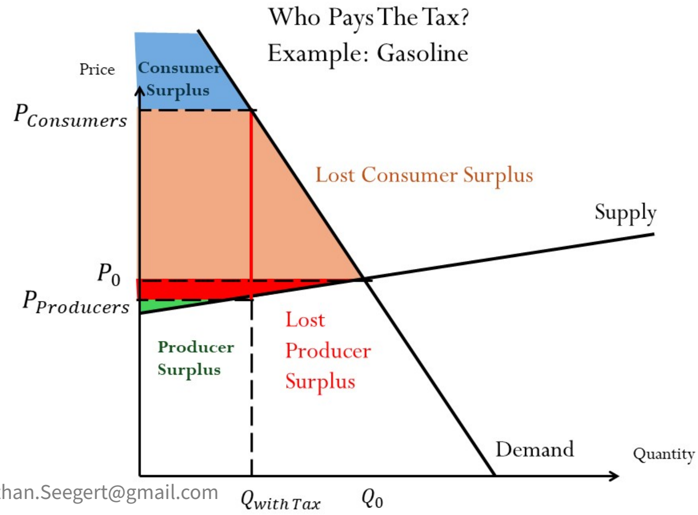
Consider a more inelastic demand



supply = demand

$$q = 100 - 9q$$

Consumers pay more when demand is more inelastic



$$\text{supply} = \text{demand}$$
$$q = 100 - 9q - t$$
$$10q = 100 - t$$
$$q = 10 - t/10$$
$$P_{\text{consumers}} = 10 + 9/10t$$
$$P_{\text{producers}} = 10 - 1/10t$$

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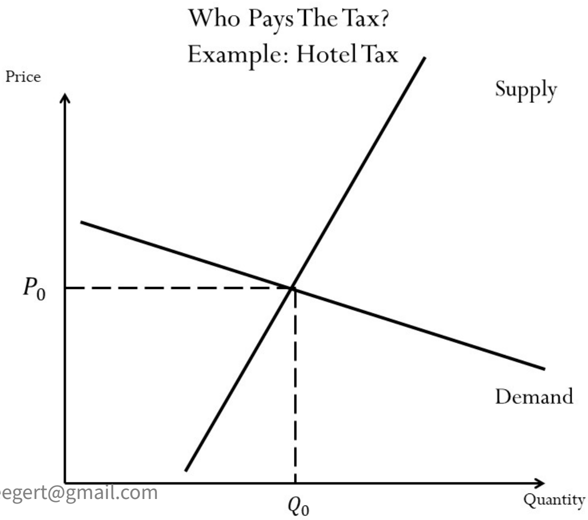
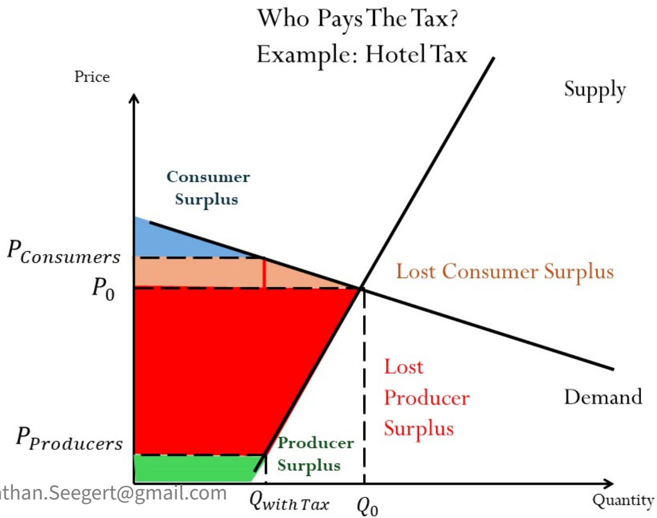
Consider a more inelastic supply



supply = demand

$9q = 100 - q$

Producers pay more when supply is more inelastic



$$\text{supply} = \text{demand}$$

$$9q = 100 - q - t$$

$$10q = 100 - t$$

$$q = 10 - t/10$$

$$P_{\text{consumers}} = 90 + 1/10t$$

$$P_{\text{producers}} = 90 - 9/10t$$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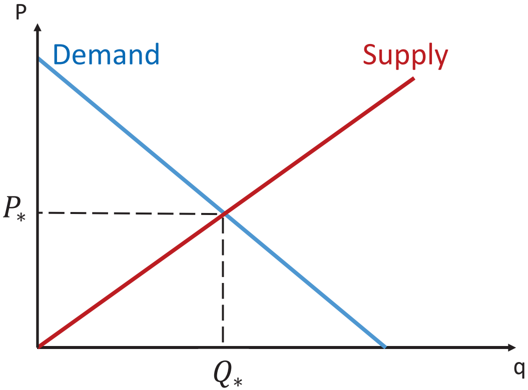
Now, let's generalize the model

The pass-through rate measures how consumer prices change.

1. $\rho = \frac{\partial P}{\partial t}$ is the pass-through rate.
2. Derive the pass-through rate using total differentiation.
3. Use the pass-through rate to do some comparative statics.

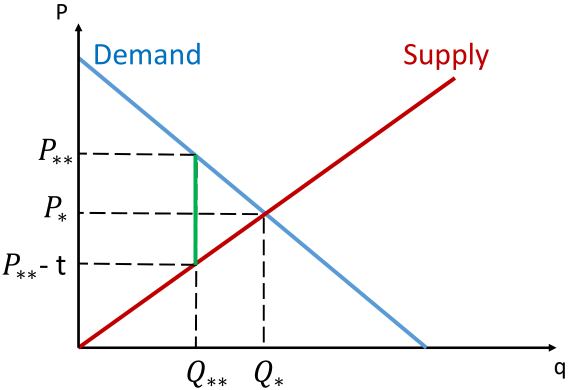
Linear supply and demand example



- Start with basic supply and demand

$$Q = D(p) = S(p)$$

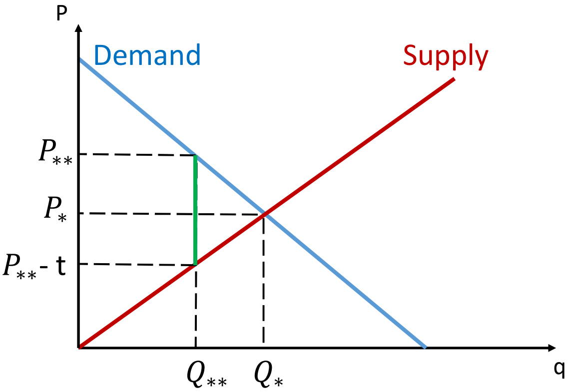
P is the price consumers pay, q = P - t is the producer price



- Add a tax

$$Q = D(P) = S(P - t)$$

The pass-through rate is the change in consumer price



- Totally differentiate

$$Q = D(P) = S(P - t)$$

$$\frac{\partial D}{\partial P} dP = \frac{\partial S}{\partial P} dP - \frac{\partial S}{\partial P} dt$$

$$\frac{dP}{dt} = \frac{-\frac{\partial S}{\partial P}}{\frac{\partial D}{\partial P} - \frac{\partial S}{\partial P}}$$

$$\rho = \frac{dP}{dt} = \frac{\varepsilon_S}{\varepsilon_S - \varepsilon_D} > 0$$

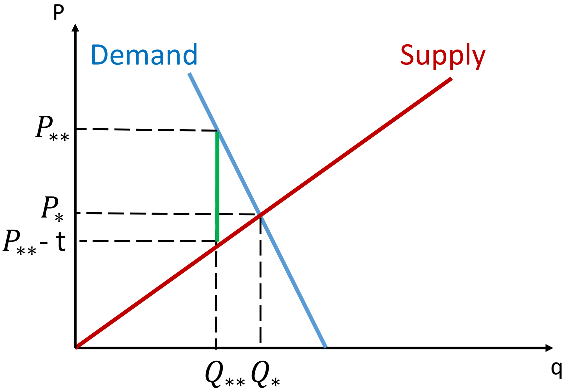
Quick reminder about elasticities

Elasticity of demand

$$\varepsilon_D = \frac{dQ}{dP}\frac{P}{Q} = \frac{dQ/Q}{dP/P} = \frac{\%\Delta Q}{\%\Delta P} = \frac{1}{slope_D}\frac{P}{Q}$$

1. Elasticity of demand is negative because the slope of the demand curve is negative.
2. With linear demand, elasticity increases in magnitude with higher P and lower Q.
3. Revenue is maximized where the elasticity of demand = -1.
4. Monopolist always in the elastic part of the demand curve $|\varepsilon_d| > 1$

The pass-through rate increases with more inelastic demand



- consumer price increases more as demand becomes more inelastic relative to supply

$$\frac{dP}{dt} = \frac{\varepsilon_S}{\varepsilon_S - \varepsilon_D}$$

- $\varepsilon_D \to 0 \to \frac{dP}{dt} = 1$

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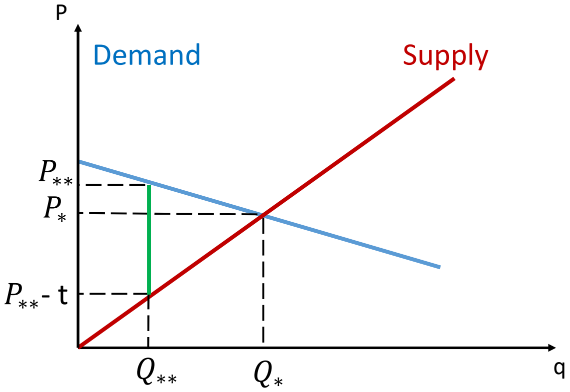
The pass-through rate decreases with more elastic demand

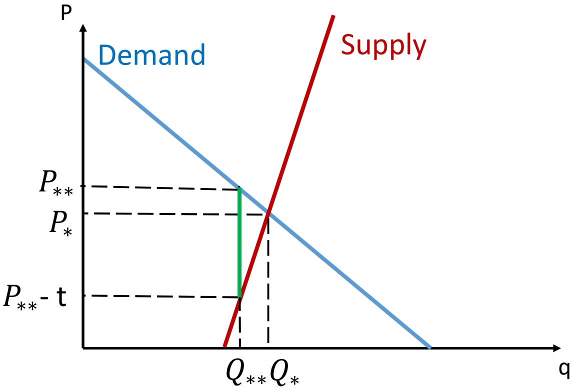


- Consumer prices increase less as demand becomes more elastic relative to supply

$$\frac{dP}{dt} = \frac{\varepsilon_S}{\varepsilon_S - \varepsilon_D}$$

- $\varepsilon_D \to \infty \to \frac{dP}{dt} = 0$

The pass-through rate decreases with more inelastic supply

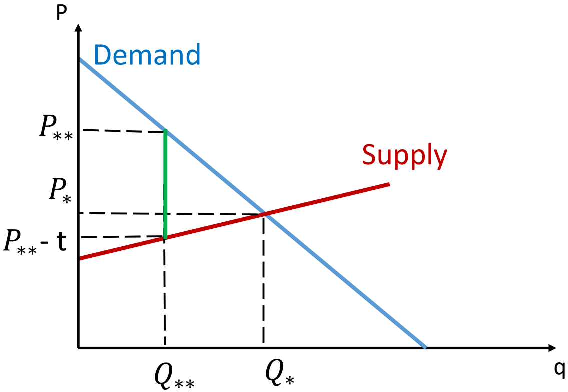


- Consumer prices increase less as supply becomes more inelastic relative to demand

$$\frac{dP}{dt} = \frac{\varepsilon_S}{\varepsilon_S - \varepsilon_D}$$

- $\varepsilon_S \rightarrow 0 \rightarrow \frac{dP}{dt} = 0$

The pass-through rate increases with more elastic supply



- Consumer prices increase more as supply becomes more elastic relative to demand

$$\frac{dP}{dt} = \frac{\varepsilon_S}{\varepsilon_S - \varepsilon_D}$$

- $\varepsilon_S \to \infty \to \frac{dP}{dt} = 1$

A.3 Can Monopolists push the tax onto consumers?

Do consumers pay more of the tax in markets where firms have more market power?

- Yes, firms with more market power, e.g., monopolists, use their market power to push the tax onto consumers
- No, firms with more market power, e.g., monopolists, already have all of the rents and thus they must pay more of the tax

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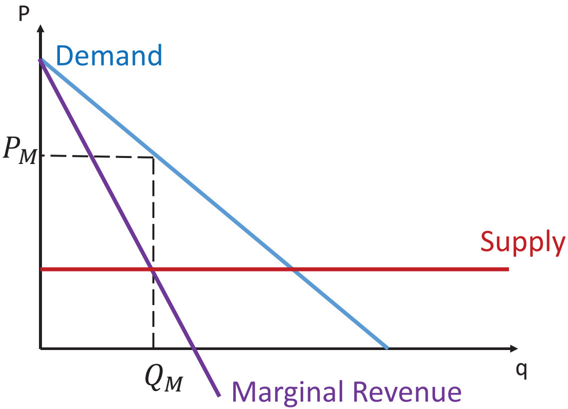
Linear supply and demand



Special case: linear demand

- Demand $P = A - BQ$
- Cost = CQ
- Monopolist problem

$$max_Q \quad (A - BQ)Q - CQ$$

$$A - BQ - BQ - C = 0$$

$$Q = \frac{A - C}{2B}, \quad P = \frac{A + C}{2}$$

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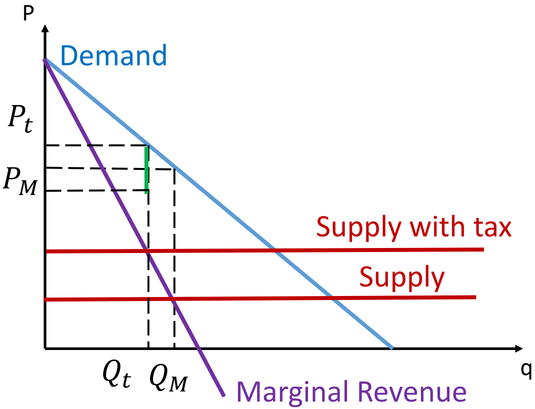
Tax with monopolist and linear supply and demand



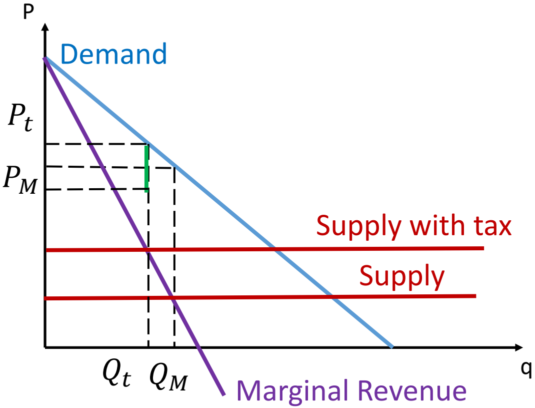
Special case: linear demand

- Demand $P = A - BQ$
- Cost = $CQ + tQ$

How much of the tax does the monopolist push to the consumer?

Note, 100 percent on consumers in competitive equilibrium

Monopolist evenly splits the tax with consumers



- Monopolist problem

$$max_Q \quad (A - BQ)Q - CQ - tQ$$

$$A - BQ - BQ - C - t = 0$$

$$Q = \frac{A - C - t}{2B}$$

$$P = \frac{A + C}{2} + \frac{1}{2}t$$

The monopolist pushes *half* of the tax onto the consumers.

More complicated examples call for other tools

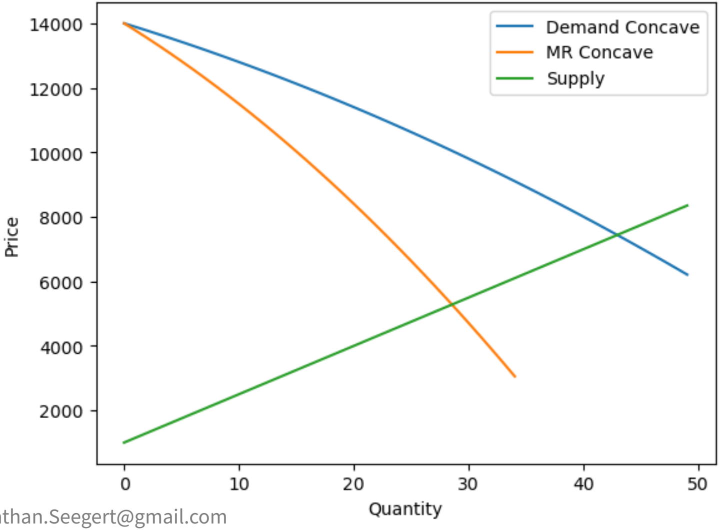
1. Linear models are nice. Easy to solve by hand.

2. Sometimes we need to go beyond linear models.

3. In these cases, there is python!
    - Or other programs like Mathematica.

4. Python code for these examples is provided on `www.nathanseegert.com/teaching`

Concave inverse demand



Concave inverse demand

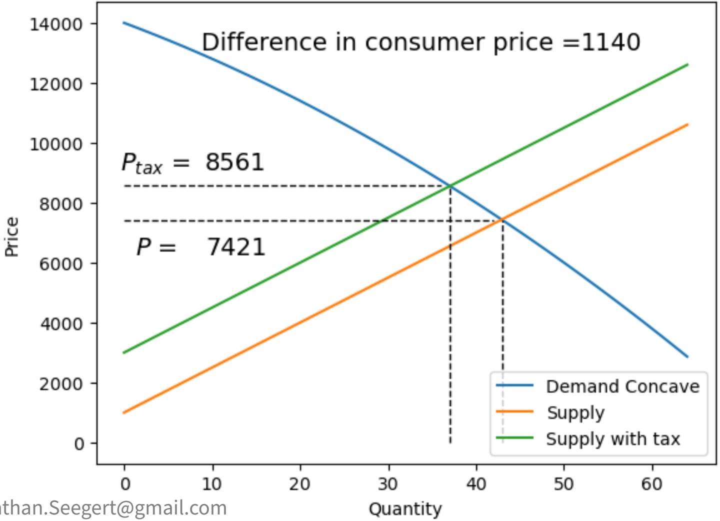
$P = A + BQ + CQ^2$

- e.g., sugar, no demand above some price.

Linear MC $C'(D(P)) = Z + YQ$

A = 14,000, B = -110, C = -1, Z = 1,000, Y = 150

Competitive price with and without a tax



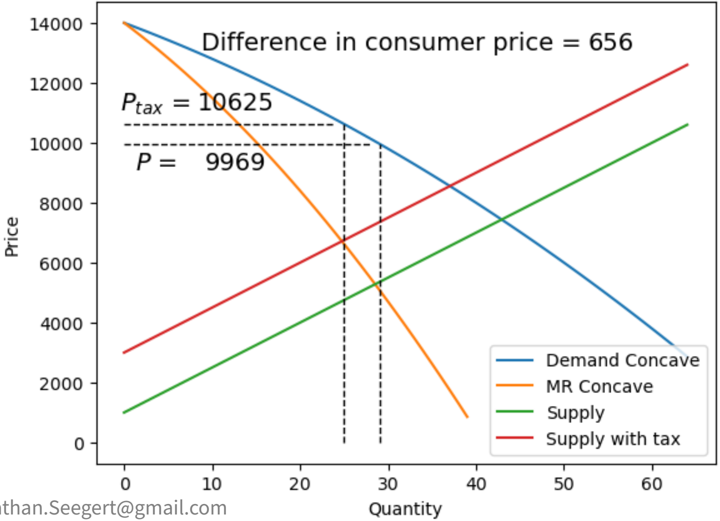
Add in a tax $t = 2000$

- Linear MC

  $C'(D(P)) = z_1 + y_1 Q + tQ$

Consumers pay 1140/2000 or 57% of the tax

Monopolist pays more of the tax with concave demand



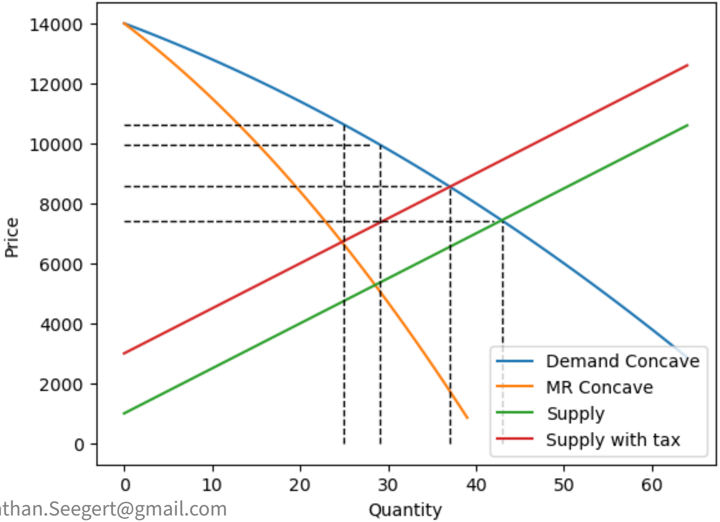
Add in a tax $t = 2000$

- Linear MC

  $C'(D(P)) = z_1 + y_1 Q + tQ$

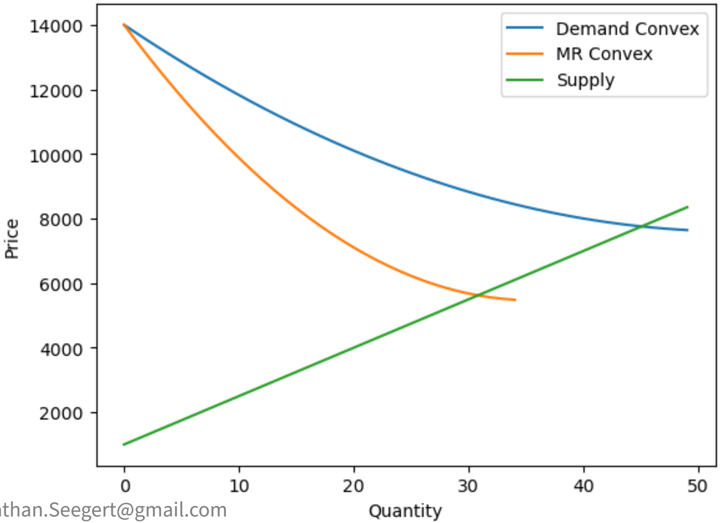
Consumers pay 656/2000 or 33% of the tax

Monopolist pays more of the tax with concave demand



- Competitive: consumers pay 1140/2000 or 57%
- Monopoly: consumers pay 656/2000 or 33%

Very convex inverse demand



Convex inverse demand

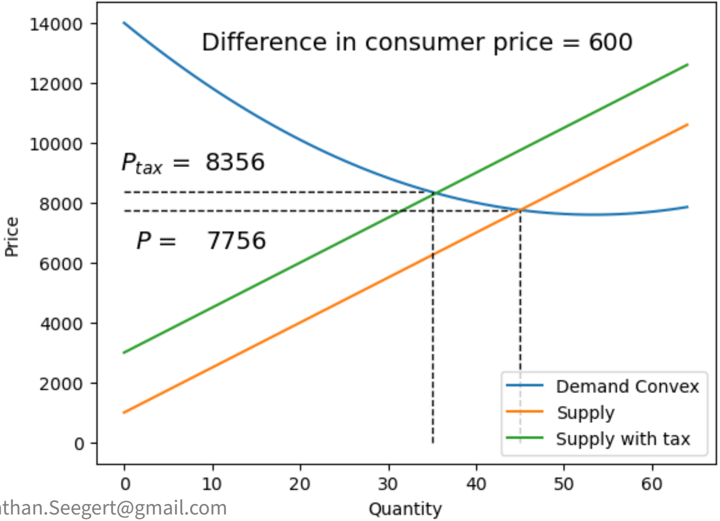
$P = A + BQ + CQ^2$

- e.g., More traditional

Linear MC $C'(D(P)) = Z + YQ$

A = 14,000, B = -240, C = 2.25, Z = 1,000, Y = 150

Competitive price with and without a tax

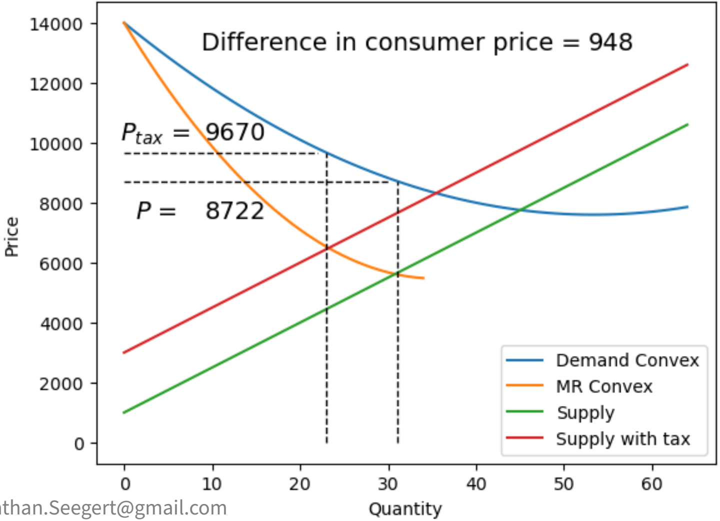


Add a tax t = 2000

- Linear MC $C'(D(P)) = z_1 + y_1 Q + tQ$

Consumers pay 600/2000 or 30% of the tax

Monopolist pays less of the tax with convex demand



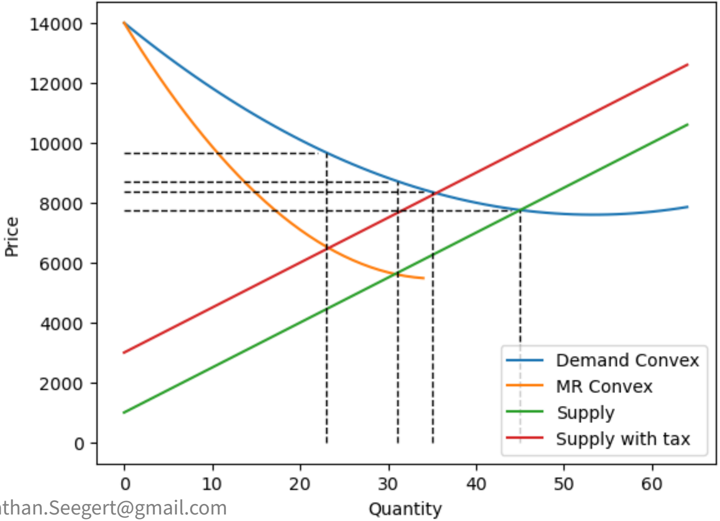
Add a tax $t = 2000$

- Linear MC

  $C'(D(P)) = z_1 + y_1 Q + tQ$

Consumers pay 948/2000 or 47% of the tax

Monopolist pays less of the tax with convex demand



Competitive: consumers pay 600/2000 or 30%

Monopoly: consumers pay 948/2000 or 47%

- Demand curve curls up faster.

Pass-through for a monopolist

How much do consumer prices change with the tax.

$$\rho = \frac{dP}{dt} = \frac{1}{1 - \frac{\varepsilon_D + 1}{\varepsilon_S} + \frac{1}{\varepsilon_{ms}}}$$

- ε_{ms} is the elasticity of marginal surplus, measuring the curvature of the demand curve.
  - $ms = -(\partial p / \partial q) q$
  - Log-concave demand $\frac{1}{\varepsilon_{ms}} > 0$.
  - log-convex demand $\frac{1}{\varepsilon_{ms}} < 0$.
- Linear demand $\varepsilon_{ms} = 1$.
- Exponential demand $1/\varepsilon_{ms} \to 0$.
- Constant elasticity demand $\varepsilon_{ms} = -\varepsilon_D$.

Pass-through for a monopolist special cases

How much do consumer prices change with the tax.

$$\rho = \frac{dP}{dt} = \frac{1}{1 - \frac{\varepsilon_D + 1}{\varepsilon_S} + \frac{1}{\varepsilon_{ms}}}$$

- Constant marginal cost ($\varepsilon_S \to \infty$) and linear demand ($1/\varepsilon_{ms} = 1$)

$$\rho = \frac{1}{2}$$

- Constant marginal cost ($\varepsilon_S \to \infty$) and concave demand ($1/\varepsilon_{ms} > 1$)

$$\rho \in \left[0, \frac{1}{2}\right]$$

- Constant marginal cost ($\varepsilon_S \to \infty$) and convex demand ($1/\varepsilon_{ms} < 1$)

$$\rho \in \left[\frac{1}{2}, 1\right]$$

Generalized formula for imperfect competition

How much do consumer prices change with the tax.

$$\rho = \frac{dP}{dt} = \frac{1}{1 - \frac{\varepsilon_D + \theta}{\varepsilon_S} + \frac{\theta}{\varepsilon_{ms}}}$$

- $\theta \in (0, 1)$ is the conduct parameter; perfect competition $\theta = 0$ monopoly $\theta = 1$.
  - In a Cournot model with N symmetric firms $\theta = 1/N$.
- Assumes $\frac{1}{\varepsilon_\theta} = 0$. See Weyl and Fabinger (2013) for a discussion of this assumption.

Generalized formula for imperfect competition

How much do consumer prices change with the tax as market power changes?

$$\frac{\partial \rho}{\partial \theta} = \frac{1}{\left(1 - \frac{\varepsilon_D + \theta}{\varepsilon_S} + \frac{\theta}{\varepsilon_{ms}}\right)^2} \left(\frac{-1}{\varepsilon_S} + \frac{1}{\varepsilon_{ms}}\right)$$

1. Sign depends on how big or small elasticity of marginal surplus, which measures the curvature of the logarithm of demand.

Incidence and market power—an empirical question

We saw examples where incidence increases or decreases with market power.

Here, the linear example can build intuition but is not general.

Seems like an interesting thing to estimate empirically.

A.4 Extensions: salience, overshifting, evasion, and empirical considerations

Including salience in the model

Consider the case in Bradley and Feldman (2020) where consumers have demand for a good with an ad valorem tax t.

- Consumers demand $x = x(p, t)$
- Consumer demand should only depend on tax-inclusive price $x = x(p(1 + t), 0)$
- Price elasticity of demand should equal gross-of-tax elasticity

$$\varepsilon_{x,p} = -\frac{\partial logx}{\partial logp} = -\frac{\partial logx}{\partial log(1 + t)} = \varepsilon_{x,1+t}$$

Including salience in the model

Bradley and Feldman (2020) conjecture that consumers under-react to less salient taxes due to inattention

$$\varepsilon_{x,p} > \varepsilon_{x,1+t}$$

- They find a 10 percent increase in 1 + t has the same effect on demand as a 1.4 percent increase in p.

Implications of incidence with salience

1. Incidence on producer prices is attenuated.
2. No tax neutrality: statutory incidence affects economic incidence.
3. Inattention unambiguously reduces DWL without income effects.
4. Inattention may reduce or increase DWL with income effects.

Can the consumer price change more than the tax?

In the literature, this is called overshifting.

- Empirically, there have been cases where the estimates suggest consumer prices change more than the tax.
    1. Evidence of overshifting, or imprecise estimates.
    2. Overshifting in alcohol (Cook 1981; Young and Bielinska-Kwapisz 2002; Kenkel 2005)
    3. Overshifting clothing and personal care items (Poterba 1996; Besley and Rosen 1999).
    4. But could be due to price points (Conlon and Rao 2020).

Example of overshifting $\rho > 1$

How much do consumer prices change with the tax.

$$\rho = \frac{dP}{dt} = \frac{1}{1 - \frac{\varepsilon_D + 1}{\varepsilon_S} + \frac{1}{\varepsilon_{ms}}}$$

- Constant marginal cost ($\varepsilon_S \to \infty$) and convex demand ($1/\varepsilon_{ms} < 0$)

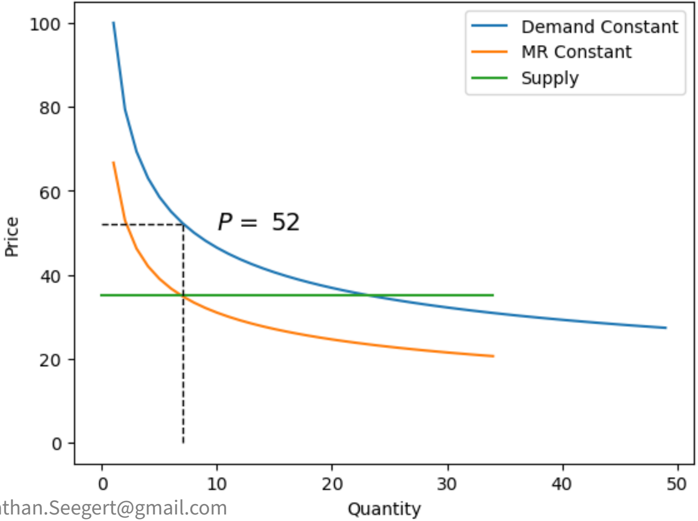
$$\rho > 1$$

- Assume constant elasticity of demand

$$Q = aP^{\varepsilon_D}$$

- The elasticity of marginal surplus $\varepsilon_{ms} = \varepsilon_D$

Monopolist pays less of the tax with convex demand



constant elasticity of demand

$$Q_D = 1000000P^{-3}$$

$$P_S = 35$$

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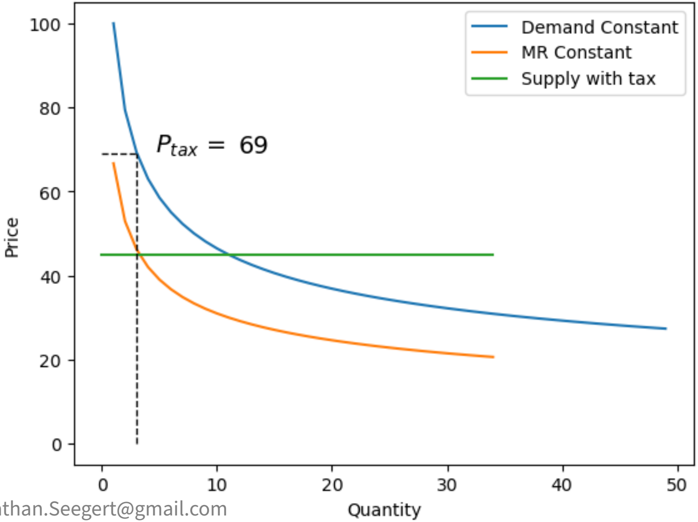
Monopolist pays less of the tax with convex demand

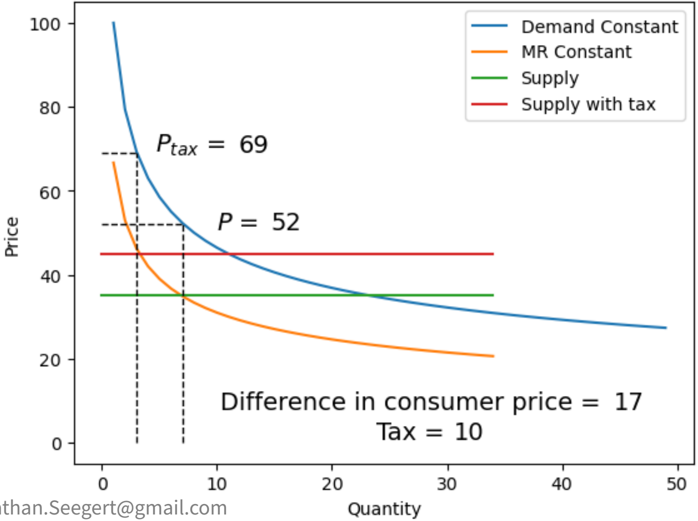


With a tax of 10

$$Q_D = 1000000P^{-3}$$

$$P_S = 35 + 10$$

Monopolist pays less of the tax with convex demand



Consumers pay 17/10 or 170% of the tax

Implications of overshifting

- In simple models, overshifting is only possible with market power.

1. Pless and van Benthem (2019) suggest using over shifting as a test for market power.
2. Agrawal and Hoyt (2019), however, show overshifting can be found empirically with perfect competition when there are multiple produces and interdependencies.

Other cases

- With multiproduct firms consumer price can *decrease* with a unit tax.
  1. Edgeworth tax paradox.
  2. Ritz (2014) show a unit tax can decrease price and industry output increases.
  3. A Pigouvian emissions unit tax can lead to an increase in industry emissions.
- Kopczuk et al. (2013) shows that with evasion a tax can have a smaller impact on prices and quantities.

Empirical estimates

We often make simplifying assumptions when going to the data.

1. Perfect competition—simplifies incidence formula.
2. Perfectly elastic supply—often no data on this.

How important are these assumptions in practice?

- Mace, Patel, and Seegert (2020) considers these assumptions using data in the marijuana market.

In the marijuana industry, as market power increases, do you suspect consumers will pay more or less

- Consumers pay more with market concentration
- Consumers pay less with market concentration

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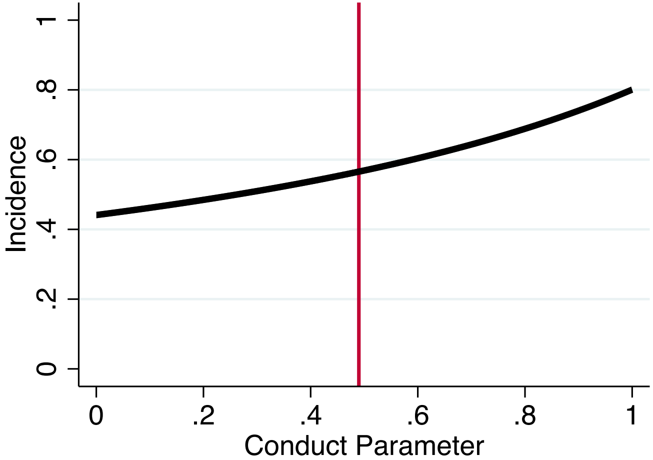
Consumers pay more of the tax with market power



Implies convex demand.

Consumers go from paying less than half to almost 80% of the tax as markets go from being competitive to monopoly.

Incidence—ripe for empirical estimation

The incidence of a tax depends on many factors.

1. Market power—in an ambiguous way.
2. Curvature of the demand function.
3. Presence of inattention and evasion.
4. Elasticities of supply and demand.

Session 1 Part B

Modeling Techniques through Models of Corporate Taxation

Session 1

B Foundation of corporate models

1. Foundation of corporate models, Fisher's separation theorem (Fisher, 1930)
2. Two-period model (Modigliani and Miller, 1958)
3. Corporate taxes

B.1 Foundation of corporate models

Fisher’s separation theorem

Player: Robinson Crusoe

Robinson Crusoe

X amount of wheat

Strategies: Consume now or invest; choose C_1, K

Stock X

Consume now C_1

Plant now K

Consume Tomorrow
C_2 = f(K)

Wheat can be consumed or planted

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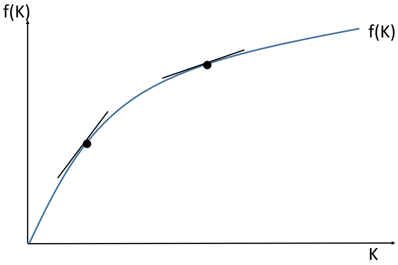
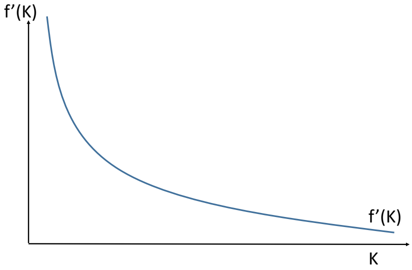
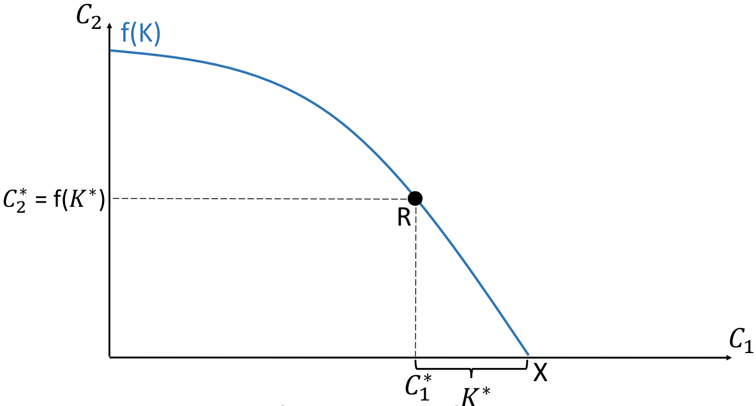
Assumptions on the production function





1. $f(0) = 0$, no production without some planting.
2. $f'() > 0$, the more you plant the more yield.
3. $f''() < 0$, the more you plant the lower the marginal yield.
   - **Diminishing returns** only so much room on the island, as you plant more use worse land or over crowd the wheat such that doubling the seed will not double the yield.

Transformation from C_1 to C_2



- Diminishing returns, get less C_2 for each unit of K as K increases.

Payoffs: utility over consumption

$$max_{C_1,C_2,K} \; U(C_1, C_2)$$

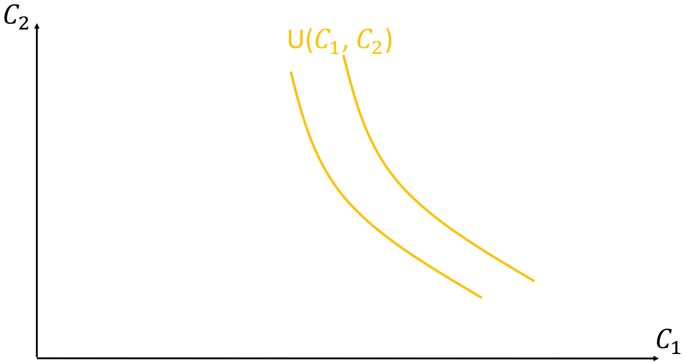
Constraints

1. Cannot consume more today than you have $0 \leq C_1 \leq X$.
2. The sum of consumption today and investment cannot be more than you have $K + C_1 \leq X$.
3. What you consume tomorrow is the yield from production $C_2 = f(K)$.

Assumptions

1. $\partial U(C_1, C_2)/\partial C_1 \equiv U_1 > 0$.
2. $\partial U(C_1, C_2)/\partial C_2 \equiv U_2 > 0$.

Indifference curves are combinations of C_1 and C_2. with the same utility



Household maximization

$$max_{C_1,C_2,K} \;\; U(C_1, C_2) \quad s.t. \;\; 0 \leq C_1 \leq X \;\; \& \;\; X = K + C_1 \;\; \& \;\; C_2 = f(K)$$

$$\mathcal{L} = U(C_1, C_2) + \lambda(X - C_1 - K) + \gamma(f(K) - C_2)$$

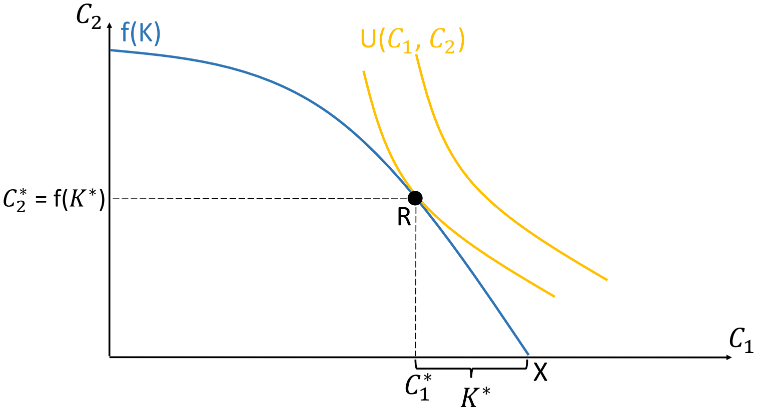
Household maximization

$$\begin{aligned}
\mathcal{L} =& U(C_1, C_2) + \lambda(X - C_1 - K) + \gamma(f(K) - C_2) \\
& \frac{\partial \mathcal{L}}{\partial C_1} : U_1 = \lambda \\
& \frac{\partial \mathcal{L}}{\partial C_2} : U_2 = \gamma \\
& \frac{\partial \mathcal{L}}{\partial K} : \lambda = \gamma f'(K) \\
& \rightarrow U_1 = \gamma f'(K) = U_2 f'(K)
\end{aligned}$$

- First-order condition: $U_1 / U_2 = f'(K)$.

Household maximization

- First-order condition: $U_1/U_2 = f'(K)$.



Optimization can be interpreted in two ways:

1. Marginal change in utility between C_1 and C_2 must equal the marginal change in production. (Marginal rate of substitution equals the marginal rate of transformation MRS = MRT)
   - $U_1/U_2 = f'(K)$.
2. The rate of time preferences $\gamma(C_1, C_2) \equiv -U_1/U_2 - 1$ equals the net marginal product of capital
   - $\gamma(C_1, C_2) = f'(K) - 1$.

In the basic model, investment depends on consumption preferences

This model tells us about the tradeoff between consumption today and tomorrow.

- What are the implications of this for corporate investment models?

How important is it for us to model consumer behavior to understand corporate investment decisions?

- Very important
- Somewhat important
- Somewhat not important
- Not important

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The role of the capital market

- Now, lets see how capital markets change the tradeoff between consumption and investment.

1. Player: a single household with endowment X.
2. Strategies:
   - Consumption now C_1,
   - Borrowing or saving B at interest rate r,
   - Investment K,
3. Payoffs: utility over consumption today and tomorrow $U(C_1, C_2)$

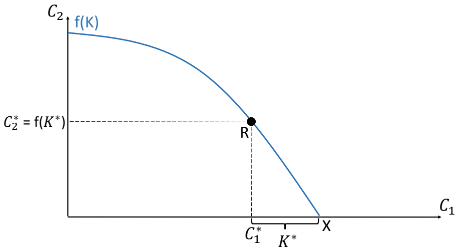
$$C_1 = X - K + B$$

$$C_2 = f(K) - (1 + r)B.$$

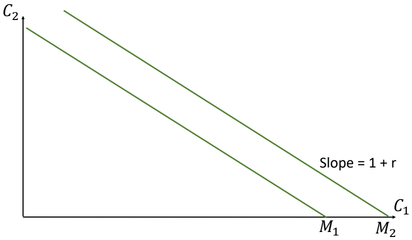
Two ways of transforming C_1 and C_2

Production/Investment:

$C_2 = f(K) = f(X - C_1).$

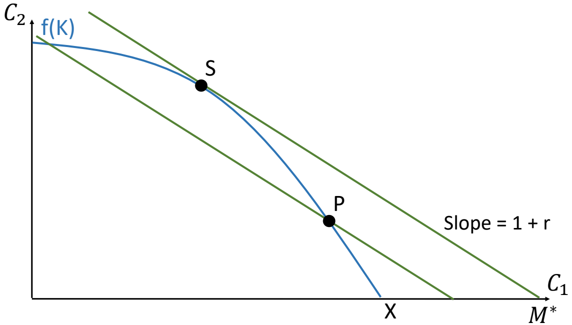


Capital markets: $C_2 = -(1 + r)C_1 + (1 + r)X.$



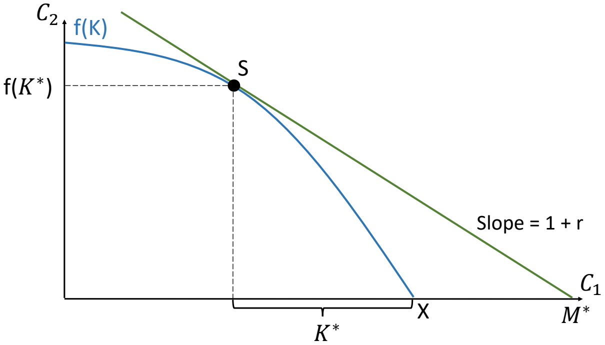
- Wealth is greater at M_2 than M_1.

First, find how much to produce (S or P)



- Maximize wealth M^*, where $f'(K) = 1 + r$

First, find how much to produce



- K^* determines how much to produce $f(K^*)$.
- M^* determines wealth.

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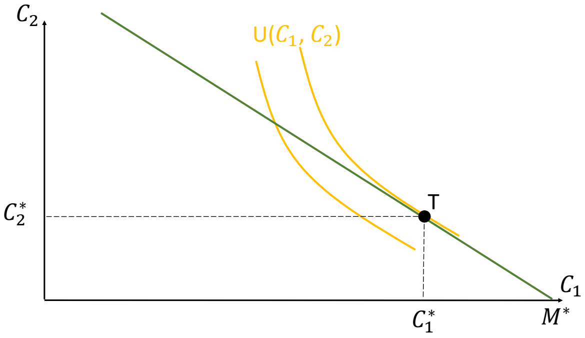
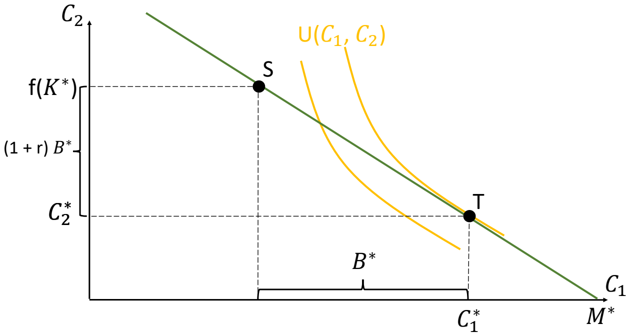
Second, find how much to consume



- Maximize utility where $U_1(C_1, C_2)/U_2(C_1, C_2) = 1 + r$, where M^* is given.

Second, find how much to consume



- Start at point S.
- Borrow B^* and repay $B^*(1 + r)$ to get to T.

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Optimization with capital markets

$$max_{C_1} \;\; U(C_1, C_2) \quad s.t. \;\; 0 \leq C_1 \leq X \;\; \& \;\; C_1 = X - K + B \;\; \& \;\; C_2 = f(K) - (1 + r)B$$

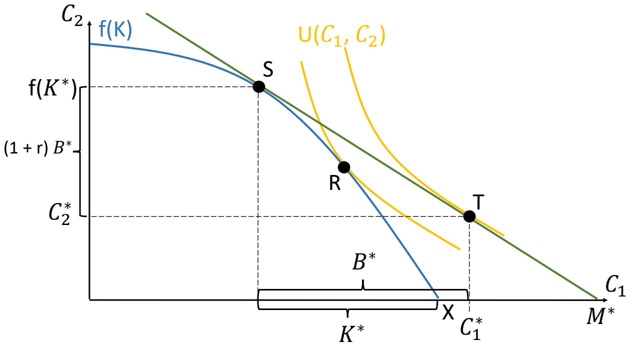
Optimality conditions for an interior solution

1. $U_1 / U_2 = 1 + r$
2. $f'(K) = 1 + r$

Marginal rate of substitution and marginal product of capital has to equal $1 + r$.

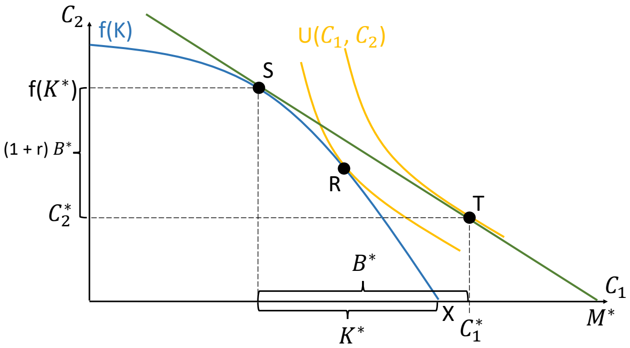
Optimization with capital markets

1. **Separation Theorem:** Point *S* defines the production decision and is independent of household preferences and initial capital endowment.



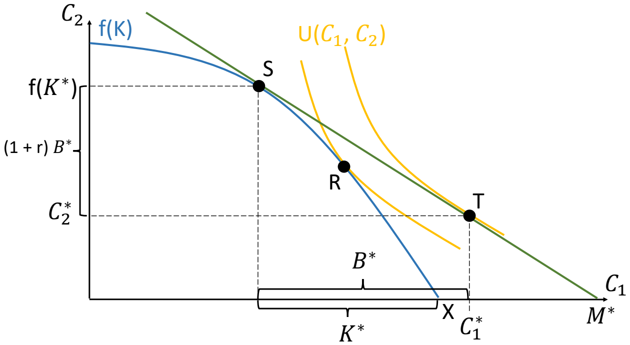
Optimization with capital markets

2. The optimal production decision maximizes wealth M^*.



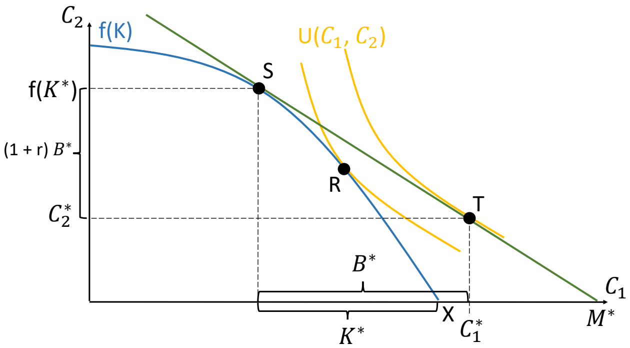
Optimization with capital markets

3. Point T defines the consumption decision and is independent of production, once we know M^*.



Capital markets expand the feasible points

4. Utility at point T is greater than at point R, and is a Pareto optimum.



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Fisher's model implications

1. **Separation theorem** The production decision is independent of household preferences and initial capital endowment.
   - $f'(K) - 1 = r$.
2. The optimal production decision maximizes wealth and net present value.
   - Wealth $M^* = \frac{f(K)}{1+r} + C_1 - B$.
   - Net present value = $\frac{f(K)}{1+r} - K$.
3. The optimal consumption decision depends on wealth.
   - Production and interest rate only matter as it impacts wealth.
4. This equilibrium is a Pareto optimum
   - No two households could make a mutually beneficial trade.
   - Aggregate production is maximized.
   - No one's utility could be increased without decreasing someone elses.

What model extensions should we consider?

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Join by QR code
Scan with your camera app

Extensions of the model

All four results hold even if there are

1. More than two periods.
2. Different capital and consumption goods.
3. Joint ownership of production across households.

This analysis is partial equilibrium

1. It holds fixed r.
2. It is poorly suited to study intertemporal allocations.
3. Solow (1956) model can be incorporated to study capital accumulation.
4. Overlapping generation models Carmichael (1982), Barro (1974), Burbidge (1963), some inconsistency of laissez-faire allocation and social planner.

B.2 Two-period model example with Modigliani and Miller (1958)

How familiar are you with Modigliani and Miller (1958)

- Very familiar
- Somewhat familiar
- I have heard of this before
- I have never heard of this before

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We want to investigate optimal debt issuances

1. How do we build a model to investigate debt issuances?

2. What is the minimum structure needed to gain insights into this problem?

3. What is the key **tradeoff**?
   - Benefit: debt can increase capital.
   - Benefit: debt can increase dividends.
   - Cost: pay back with interest next period.

Cost and benefit of debt

B is debt (bonds, borrowing).

1. Benefit: debt can increase capital or dividends.

$$B = K + D - X$$

   - K is capital (investment) used to produce $f(K)$.
   - D is dividends (what we consume now).
   - X is initial cash on hand (exogenously given).

2. Cost: pay back with interest next period.

$$(1 + r)B/(1 + r)$$

   - Pay back $(1 + r)B$, but do so next period, r is the interest rate.

Basic model moving forward relabeled dividends and debt

A firm chooses its dividend and debt policies to maximize the value of the firm, which is consumption today plus discounted consumption tomorrow:

$$max_{B,D} \quad D + \frac{f(K) - (1+r)B}{1+r} = D + \frac{f(X+B-D) - (1+r)B}{1+r}$$

1. B is debt.
2. Capital is $K = X + B - D$.
3. D is dividends (what we consume now).
4. X is initial cash on hand (exogenously given).
5. r is the interest rate.

Marginal benefit equals marginal cost

A firm chooses its dividend and debt policies to maximize the value of the firm

$$max_{B,D} \quad V = D + \frac{f(X + B - D) - (1 + r)B}{1 + r}$$

First order condition with respect to debt B

First order condition with respect to dividends D

Marginal benefit equals marginal cost

A firm chooses its dividend and debt policies to maximize the value of the firm

$$max_{B,D} \quad V = D + \frac{f(X + B - D) - (1 + r)B}{1 + r}$$

First order condition with respect to debt B

$$\partial B: \quad \frac{f'(X + B - D)}{1 + r} - \frac{1 + r}{1 + r} = 0$$

$$\underbrace{f'(X + B - D)}_{\text{marginal benefit}} = \underbrace{1 + r}_{\text{marginal cost}}$$

Marginal benefit equals marginal cost

Firm chooses its dividend and debt policies to maximize the value of the firm

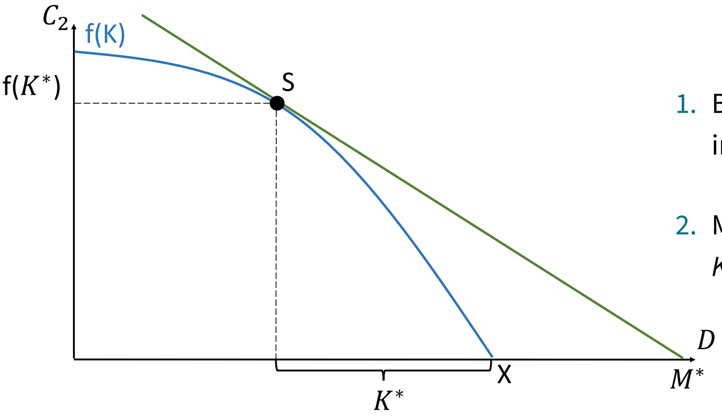
$$max_{B,D} V = D + \frac{f(X + B - D) - (1 + r)B}{1 + r}$$

First order condition with respect to dividends D

$$\partial D : \quad 1 - \frac{f'(X + B - D)}{1 + r} = 0$$

$$\underbrace{f'(X + B - D)}_{\text{marginal benefit}} = \underbrace{1 + r}_{\text{marginal cost}}$$

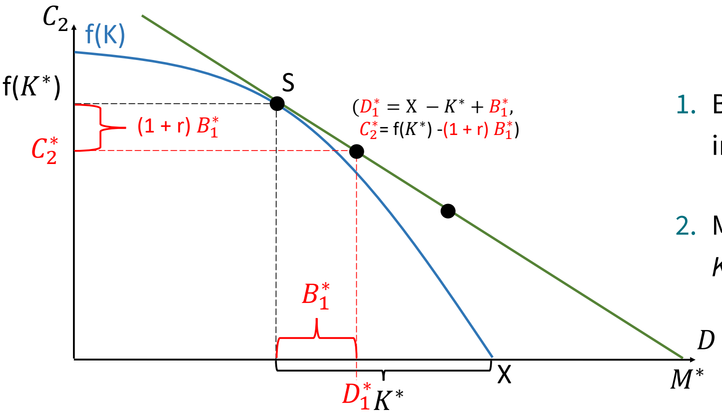
Capital is determined but not dividends or debt



1. Both first-order conditions imply $f'(K) = 1 + r$.

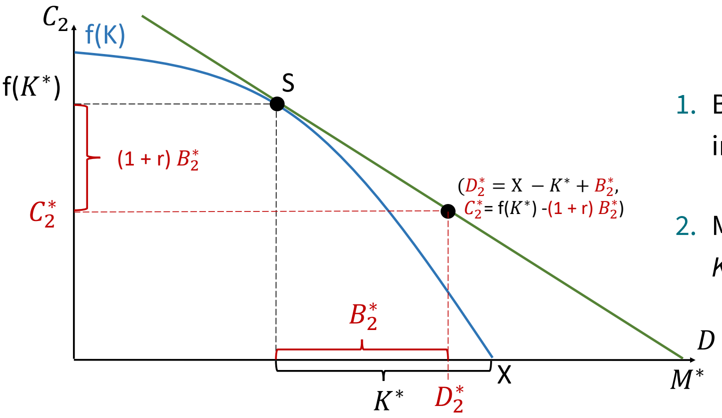
2. Many ways of getting the same $K = X + B - D$.

Borrow a little to fund modest dividends



1. Both first-order conditions imply $f'(K) = 1 + r$.
2. Many ways of getting the same $K = X + B - D$.

Borrow a lot to fund a large dividend



1. Both first-order conditions imply $f'(K) = 1 + r$.
2. Many ways of getting the same $K = X + B - D$.

Modligani-Miller in our basic model

1. The optimal debt and dividend policies are indeterminate!
2. Value remains constant with an increase in debt and higher dividend payments (or the reverse).
3. Of course, this is not the end of story because there are taxes.

B.3 Corporate taxes

Do corporate taxes distort investment decisions?

How distortionary are corporate taxes to firm investment?

- Very distortionary
- Somewhat distortionary
- Barely distortionary
- Not distortionary at all

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Adding corporate taxes to our basic model

We want to investigate whether/how corporate income taxes distort investment.

1. Consider investment from equity issuances E and the tradeoff between today and tomorrow:
   - Cost: $-E$ today.
   - Benefit: higher profits tomorrow $f(X + E)$, where $K = X + E$.

Setup the objective function

Adding corporate taxes to our basic model with equity

Shareholders choose equity E to maximize value V, by trading off less income now with higher profits tomorrow.

$$max_E \quad V = -E + \frac{(1-\tau_c)f(X+E)}{1+r}$$

Does the corporate income tax τ_c distort this tradeoff for firms?

Take the first-order condition

Adding corporate taxes to our basic model with equity

Shareholders choose equity E to maximize value V, by trading off less income now with higher profits tomorrow.

$$max_E \quad V = -E + \frac{(1-\tau_c)f(X+E)}{1+r}$$

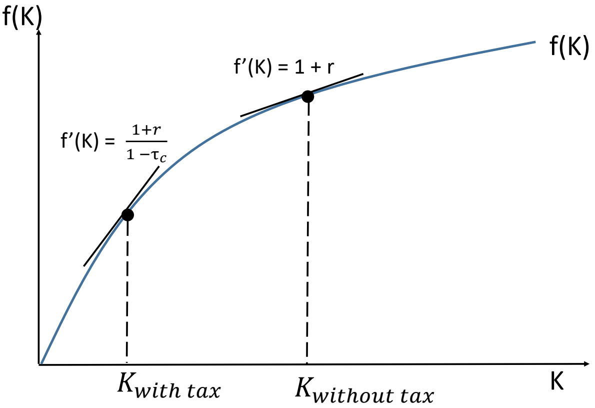
Take the first-order condition

$$\partial E: \quad -1 + \frac{(1-\tau_c)f'(K)}{1+r} = 0$$

$$\rightarrow f'(K) = \frac{1+r}{1-\tau_c}$$

- Corporate taxes have a large distortion!

Distortions to investment from corporate taxes and equity



Adding corporate taxes to our basic model with debt financing

2. Investment could come from debt B that creates a tradeoff between more production tomorrow and payment with interest tomorrow:
   - Cost: $(1 + r)B$ tomorrow.
   - Benefit: higher profits tomorrow $f(X + B)$.

Does the corporate income tax τ_c distort this tradeoff for firms?

- Let $\gamma \in [0, 1]$ be the percent of debt costs that are tax deductible.

Write down this two-period model.

Corporate taxes in the model with debt investment

Shareholders choose B to maximize firm value trading off higher profits and more debt

$$max_B \qquad V = \frac{(1-\tau_c)\left[f(X+B) - \gamma(1+r)B\right] - (1-\gamma)(1+r)B}{1+r}$$

- Let $\gamma \in [0,1]$ be the percent of debt costs that are tax deductible.

Take the first-order condition.

Corporate taxes in the model with debt investment

Take the first-order condition

$$\partial B: \quad \frac{(1-\tau_c)\left[f'(K)-\gamma(1+r)\right]-(1-\gamma)(1+r)}{1+r}=0$$

$$\rightarrow f'(K)=\gamma(1+r)+(1-\gamma)\frac{1+r}{1-\tau_c}$$

- If $\gamma = 1$, then there is no distortion from corporate taxes if debt is the marginal source of investment.
- If $\gamma = 0$, then there is a large distortion of corporate taxes (Hall and Jorgenson, 1967).

What else might be important in this model?

1. Depreciation schedules for tax purposes relative to economic depreciation.

2. How would we empirically test whether corporate taxes distort investment or taxable income?

What is the elasticity of corporate taxable income in the US? For a 1% change in the net of tax rate (1 -t), tax revenue changes by X %

- 0.1
- 0.25
- 0.5
- 0.75
- 1
- 1.25
- 1.5
- 1.75
- 2
- More than 2

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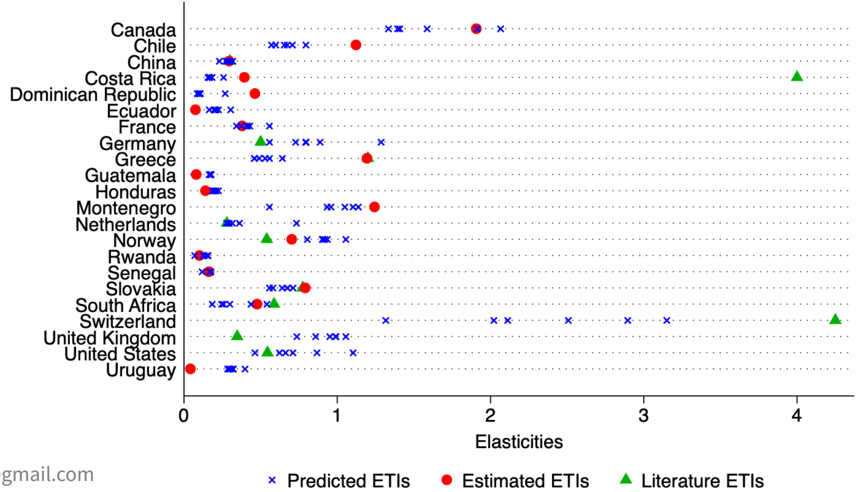
Empirical estimates—How elastic are firms?

Use changes in tax rates from tax schedules (bunching).

- Gruber and Rauh (2007); Coles, Patel, Seegert, and Smith (2021); Dwenger and Steiner (2012); Lediga, Riedel, and Strohmaier (2019); Krapf and Staubli (2020); Bukovina, Lichard, Palguta, and Zudel (2021); Bachas and Soto (2021); Massenz and Bosch (2022).

What is the elaticity of corporate taxable income with respect to the net of tax rate?

Empirical estimates of the distortions of corporate taxes



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Session 2

Modeling Techniques through Models of Corporate Taxation

Session 2

A. Adding features to the basic model
   1. Dividend taxes (new vs old view) (Chetty and Saez, 2005; Ohrn and Seegert, 2019)
   2. Agency problems and risk with taxes (Bennett et al., 2020; Arnemann et al., 2022)
   3. Mergers and acquisitions (Coles, Sandvik, and Seegert, 2020)
   4. Tax evasion (Patel and Seegert, 2020)
B. Connecting the model with empirical work
   1. Partial and general equilibrium
   2. Envelope theorem
   3. Sufficient statistics
   4. Structural parameter estimation

A.1 Dividend taxes

Do dividend taxes distort investment?

Do dividend taxes distort internal investment decisions of firms?

- Yes
- No
- Sometimes

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Do dividend taxes distort investment behavior?

Firms choose dividends and equity policy D and E, to maximize firm value by trading off dividends or equity today and production tomorrow.

$$V = D - E + \frac{f(X - D + E) + X - D + E}{1 + r}$$

- Today firms can pay D dividends or ask for equity E.
- Tomorrow capital $K = X - D + E$ produces $f(K)$ and the firm liquidates and gives back K.[1]

[1] This is important because of rules on dividend taxes between equity and retained earnings (Chetty and Saez, 2010).

Do dividend taxes distort investment behavior?

Dividend taxes make dividends less valuable today, maybe firms will **over-invest**. But maybe not?

$$V = (1 - \tau_d)D - E + \frac{(1 - \tau_d)[(1 - \tau_c)f(X - D + E) + X - D] + E}{1 + r}$$

- Dividend taxes τ_d are paid on dividends today, but not rebated to equity.
- Dividend taxes paid on production and retained earnings tomorrow, but not equity.
- For comparison, model corporate income tax τ_c.

Two cases: issue equity or pay a dividend, not both

We consider the objective function in two cases.

1. Consider optimization when the firm issues equity.
2. Consider optimization when the firm issues dividends.
3. Do we expect a difference?

Case one: firm issues equity

Let $D = 0$, and firms choose equity E to maximize firm value.

$$max_E \quad V = -E + \frac{(1-\tau_d)[(1-\tau_c)f(X+E)+X]+E}{1+r}$$

Case one: firm issues equity

Let $D = 0$, and firms choose equity E to maximize firm value.

$$max_E \quad V = -E + \frac{(1-\tau_d)[(1-\tau_c)f(X+E)+X]+E}{1+r}$$

Take the first-order condition

$$\partial V/\partial E = -1 + \frac{(1-\tau_d)[(1-\tau_c)f'(X+E)]+1}{1+r} = 0$$

$$f'(X+E) = \frac{r}{(1-\tau_d)(1-\tau_c)}$$

- Dividend tax rate distorts investment similar to corporate taxes.

Case two: firm pays a dividend

Let $E = 0$, and firms choose dividends D to maximize firm value.

$$max_D \qquad V = (1-\tau_d)D + \frac{(1-\tau_d)[(1-\tau_c)f(X-D) + X - D]}{1+r}$$

Take the first-order condition and determine how big the distortion from the dividend tax is.

Case two: firm pays a dividend

$$\partial V / \partial D = (1 - \tau_d) - \frac{(1 - \tau_d)[(1 - \tau_c)f'(X - D) + 1]}{1 + r} = 0$$

$$(1 - \tau_c)f'(X - D) + 1 = \frac{(1 - \tau_d)(1 + r)}{(1 - \tau_d)}$$

$$f'(X - D) = \frac{1 + r}{(1 - \tau_c)}$$

- Dividend tax rate drops out—no distortion.

New view vs old view—matter of firm type

Whether dividend taxes distort investment decisions seem to depend on whether the firms are issuing equity or paying dividends.

1. Old view: **distortion**. Cash constrained firms; $D = 0$ and $E > 0$,
2. New view: **no distortion**. Cash rich firms; $D > 0$ and $E = 0$,
3. Cash intermediate firms; $D = 0$ and $E = 0$.
   - Ignore because not that interesting.

New view vs old view—empirical evidence

- Chetty and Saez (2005) document
  1. Dividends increased after the dividend tax cut of 2003.
     - Seems at odds with new view.
  2. The adjustment was rapid.
     - Seems at odds with old view, because supply mechanism would take longer.
- Gordon and Dietz (2008) and Chetty and Saez (2010) propose an agency model based on Jensen and Meckling (1976).
- Yagan (2015) finds that despite increased dividend payments there was no change to corporate investment or employee compensation.
  - Consistent with the new view—but a puzzle, where did the money come from?
- Ohrn and Seegert (2019) include M&A into the model and show it reconciles all of the empirical findings.
  - The model is also consistent with evidence on M&A behavior around 2003.

A.2 Agency problems and risk with taxes

Expected Utility with CARA utility

What agency problems may exist between CEOs and shareholders?

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Adding in principal-agent concerns into the model

1. We want to investigate agency problems between managers and stockholders (Jensen and Meckling, 1976; Smith and Stulz, 1985).

2. Consider two potential agency problems
   - Different incentives (e.g., empire building) for the manager.
   - Different risk preferences for the manager (e.g., risk averse).

Empire building for managers

A large amount of literature suggests that managers want to grow.

- This could be for empire building.
- This could be pet projects.
- This could be for compensation comparisons.

We model this as adding in $g(K), g'(K) > 0, g''(K) < 0$. to the manager's objective function, where K is capital.

Differences in risk preferences

Managers are risk averse while shareholders are risk neutral, e.g., CARA utility.

$$U(w_0, K, V) = -e^{-\rho(w_0+\alpha V(K)+g(K))}.$$

- ρ risk aversion parameter.
- w_0 external wealth.
- $V(K)$ is firm value that is uncertain (normal or log-normally distributed).
- α weight that firm value enters the manager's utility.

Expected utility with mean and variance

We want to capture risk preferences, and so we assume the manager's utility is CARA or CRRA with normal or log-normally distributed uncertainty.

$$u = w_0 + \alpha\mu_V(K) - \frac{1}{2}\rho\sigma^2(K) + g(K)$$

- $\mu_V(K)$ expected value of the firm depends on investment K.
- $\sigma^2(K)$ variance of firm value, which depends on investment K.

Modeling compensation packages of managers

Now, allow shareholders to compensate managers to align incentives.

1. Effective ownership δ through accumulation of stock and options net of dispositions.
   - To account for managers having other incentives (e.g., empire building).
2. Compensation convexity through vega, ν—such as option grants.
   - To account for managers being more risk averse than shareholders.
3. Together, these features update manager's utility

$$u = w_0 + (\alpha + \delta)\mu_V - \frac{1}{2}(\rho - \nu)(\alpha + \delta)^2\sigma^2 + g(K)$$

Add in a dividend tax

1. If the government increases the dividend tax τ_d, how would compensation committees need to change δ and ν to get the same incentive alignment as before the tax change?

Agency model—objective function with dividend taxes

$$w_0 + (1 - \tau_d)(\alpha + \delta)\mu - \frac{1}{2}(\rho - \nu)(\alpha + \delta)_0^2(1 - \tau_d)^2\sigma^2$$

1. Hypothesis 1: Higher dividend taxes may require compensation committees to increase δ to get the same incentive alignment.

2. Hypothesis 2: Higher dividend taxes may allow compensation committees to decrease ν to get the same risk preference alignment.

Empirical evidence of personal taxes and CEO compensation

Using the previous model, or something similar, the following research investigates the role of taxes on firm behavior/compensation.

1. Arnemann, Buhlmann, Ruf, and Voget (2022) find higher income taxes on CEOs lowers firm performance.
2. Bennett, Coles, and Wang (2020) find income taxes are *not* paid by the CEO.
3. Coles, Sandvik, and Seegert (2020) find that personal taxes and different compensation incentives provide different incentives for M&A activity and ultimately performance.

A.3 Mergers

Do dividend taxes distort acquisitions?

What makes mergers and acquisitions different than internal investment?

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What makes mergers different than internal investment?

1. Synergies can amplify the production of a company.
2. The price for an M&A may depend on tax treatment.
3. Manager incentives, can change risk profile.

Begin with standard two-period model

1. Start with a basic two-period model of corporate decision-making.
2. Firms, maximize shareholder value V and choose their level of dividend D, such that capital in period 2 is given by their retained earnings minus dividends $I = X - D$.
3. Profits, net depreciation, from investment is given by $f(I)$ and discounted by the interest rate r.

$$max_D \qquad V = D + \frac{f(X-D) + X - D}{1+r}.$$

Add dividend taxes

Dividend taxes are paid in both periods (new view model).

$$max_D \qquad V = (1 - \tau_d)D + (1 - \tau_d)\frac{f(X - D) + X - D}{1 + r}.$$

Dividend taxes do not distort internal investment

What is an acquisition?

If the firm makes an acquisition it

1. Acquires some amount of capital C, production technology $g(\cdot)$, and potential synergies θ.
$$\theta(g(C) + C)$$
2. Pays the target firm their reservation payment*
$$M = (1 - \tau_d)\frac{g(C) + C}{1 + r}.$$

Value with an acquisition

$$(1-\tau_d)V_1 = (1-\tau_d)D_1 + (1-\tau_d)\frac{f(X-(D_1+M)) + \theta(g(C)+C) + I_1}{1+r}.$$

- Internal investment with an acquisition $I_1 = X - D_1 - M$.

When does an acquisition get done? When *should* it get done?

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When does an acquisition get done?

Managers make an acquisition when $V_1 > V_0$, or when synergies are greater than some threshold

$$(1-\tau_d)D_1 + (1-\tau_d)\frac{f(X-(D_1+M)) + \theta(g(C)+C) + I_1}{1+r} > (1-\tau_d)D_0 + (1-\tau_d)\frac{f(X-D_0)+I_0}{1+r}.$$

When does an acquisition get done?

Managers make an acquisition when $V_1 > V_0$, or when synergies are greater than some threshold

$$
\begin{aligned}
(1-\tau_d)D_1 + (1-\tau_d)\frac{f(X-(D_1+M)) + \theta(g(C)+C) + I_1}{1+r} &> (1-\tau_d)D_0 + (1-\tau_d)\frac{f(X-D_0)+I_0}{1+r} \\
\theta(1-\tau_d)\frac{g(C)+C}{1+r} &> (1-\tau_d)(D_0 - D_1) \\
\theta M &> (1-\tau_d)M \\
\theta &> (1-\tau_d)
\end{aligned}
$$

The dividend tax distorts M&A but not internal investment

Acquisitions should occur whenever $\theta > 1$.

We just found they will take place whenever,

$$\theta > (1 - \tau_d)$$

What can we learn/test from this equilibrium condition?

We can also add in agency problems

PROPOSITION 1 *An increase in the dividend tax rate has an ambiguous effect on the threshold for the acquisitions firms undertake.*

$$\frac{\partial \theta^*}{\partial \tau_d} = -1 + (\rho - \nu)\delta(1 - \tau_d)\sigma^2\gamma M \gtrless 0.$$

The simple conclusion is not so straightforward now—its an empirical question

New testable hypotheses

PROPOSITION 2 *The effect of changes in the dividend tax rate is smaller for managers with less effective risk aversion.*

$$\frac{\partial(\partial\theta^*/\partial\tau_d)}{\partial\nu} = -(1-\tau_d)\delta\sigma^2\gamma M < 0.$$

PROPOSITION 3 *The effect of changes in the dividend tax rate is smaller for managers with less effective ownership.*

$$\frac{\partial(\partial\theta^*/\partial\tau_d)}{\partial\delta} = (\rho-\nu)(1-\tau_d)\sigma^2\gamma M > 0.$$

A.4 Tax evasion

Tax enforcement policies

What percent of profits do you think are under-reported by firms in the US?

- Less than 5%
- 5%-20%
- 20%-50%
- 50%-80%
- 80%-95%

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Allingham-Sandmo Model applied to firms Patel and Seegert (2020)

1. Firms have profits W.
2. Firms decide how much income to evade E.
3. The tax authority imposes a tax rate t and a penalty rate $\theta t > t$ applied to profits if detected with probability p.
4. The firm maximizes

$$V = (1 - p)((1 - t)W + tE) + p((1 - t)W - \theta tE)$$

Take a moment and solve the model

First-order condition has E drop out

$$V = (1-p)((1-t)W + tE) + p((1-t)W - \theta tE)$$

Let's take the first-order condition

$$\frac{\partial V}{\partial E} = \underbrace{(1-p)t}_{\text{Marginal Benefit}} - \underbrace{p\theta t}_{\text{Marginal Cost}}$$

If $MB > MC$ infinite evasion, else no evasion.

Add in curvature to the objective function

Let the probability that evasion is detected increase with evasion.

- $p(E)$, where $p'(E) > 0$.
- $p(E) = \frac{E}{W}$, if $E < W$ and 1 otherwise.

The model now predicts an interior solution

Objective function.

$$V = (1 - p)tE - p\theta tE + (1 - t)W$$

First-order condition

$$\begin{aligned}
\frac{\partial V}{\partial E} &= (1 - \frac{E}{W})t - t\frac{E}{W} - \frac{E}{W}\theta t - \frac{E}{W}\theta t = 0 \\
1 &= (1 + \theta)2\frac{E}{W} \\
E &= \frac{W}{2(1 + \theta)}
\end{aligned}$$

Note, tax rate drops out, but if penalty had been θE instead of θtE it would be there.

Suppose the probability of detection is uncertain

$$V = (1 - p)tE - p\theta tE + (1 - t)W$$

- Let $p = \phi \frac{E}{W}$
- Let $\phi \sim U[0, 2]$ such that $E[\phi] = 1$.

Evasion still an interior solution

The objective function

$$E[V] = (1 - \frac{E}{W})tE - \frac{E}{W}\theta tE + (1 - t)W$$

First-order condition

$$\frac{\partial V}{\partial E} = t - 2\frac{E}{W}t - 2\frac{E}{W}t\theta = 0$$

$$1 = 2(1 + \theta)\frac{E}{W}$$

$$E = \frac{W}{2(1 + \theta)}$$

This is same as before

Now, consider a policy change

The tax authority decides whether to increase their detection technology such that $\tilde{\phi} = 2\phi$.

Suppose the tax authority only finds it beneficial to make the investment if $\phi < \alpha$.

$$E[\phi] = \begin{cases} 1 + \frac{\alpha}{2}, & \text{if } \phi > \alpha \\ \frac{\alpha}{2}, & \text{if } \phi \leq \alpha \end{cases}$$

$$E[\tilde{\phi}|\phi \leq \alpha] = 2E[\phi|\phi \leq \alpha] = \alpha$$

$$E[p|\phi \leq \alpha] = 2E[\tilde{\phi}|\phi \leq \alpha] = \alpha\frac{E}{W}$$

Evasion may go up or down with the policy

The objective function

$$E[V] = (1 - \alpha\frac{E}{W})tE - \alpha\frac{E}{W}\theta tE + (1 - t)W$$

First-order condition

$$\frac{\partial V}{\partial E} = t - \frac{E}{W}2\alpha t - \frac{E}{W}2\alpha t\theta = 0$$
$$1 = (1 + \theta)\alpha\frac{E}{W}$$
$$E = \frac{W}{2\alpha(1 + \theta)}$$

If $\alpha < 1$, then evasion goes up with the policy.

Empirically, this information updating can be important

1. Patel and Seegert (2020) find that the IRS increasing information disclosure in 2011 reduced corporate receipts by $1.3 billion.

2. Gaulin, Navarro-Sanchez, Seegert, and Yang (2020) find that people updated about risks of COVID-19 more if mask mandate imposed by their county than their state.

Session 2 Part B

Modeling Techniques through Models of Corporate Taxation

Session 2

B Modeling tools

1 Partial and general equilibrium

2 Envelope theorem

3 Sufficient statistics

4 Structural parameter estimation

B.1 Partial and general equilibrium

How does inflation distort investment?

Does inflation distort investment decisions?

- Yes
- No

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Back to the future with a model of inflation

Want to know whether inflation affects real investment.

1. Interest rates r should adjust for inflation π.
   - Irving Fisher 1930 noted nominal interest rates should rise one-for-one with inflation $dr/d\pi = 1$.
2. Interest rates affect real investment.
   - Interest rates are nominal while capital is a real variable (Darby, 1975; Feldstein, 1976).
3. Plausible that inflation, therefore, affects real investment.
4. Modeling tool:
   - Show comparative statics using total differentiation of equilibrium condition.

What do we need in our model?

1. What is the key question?
   - How/does inflation distort the tradeoff and therefore investment?
   - Inflation makes money borrowed today not as costly to payoff tomorrow.
2. What is the key tradeoff we are interested in?
   - Interested in investment.
   - Benefit is more production $f(K)$.
   - Cost is cost of investment (borrowing to look at inflation) rB.

Demand of capital

Firms choose borrowing B to maximize value taking into account inflation.

$$max_B \quad V = (f(K) - rB)(1 - \tau_c) + \tau_c \delta K - \delta K + \pi B$$

1. Capital is increasing with borrowing $K = X + B$.
2. After-tax profits net interest payments $(f(K) - rB)(1 - \tau_c)$.
3. Inflationary gains on the stock of nominal borrowing. πB.
4. Capital depreciation δK and value of tax deduction for depreciation $\tau_c \delta K$.
    - Is this last piece necessary for the model?

Take the first-order condition and determine whether investment depends on the inflation.

Intuition and next steps

Does investment depend on inflation?

What assumptions went into this finding? Is that reasonable?

In partial equilibrium inflation distorts investment

Firms choose borrowing B to maximize value taking into account inflation.

$$max_B \qquad V = (f(K) - rB)(1 - \tau_c) + \tau_c \delta K - \delta K + \pi B$$

Take the first-order condition.

$$\partial V / \partial B = (1 - \tau_c) f'(K) - (1 - \tau_c) r + \tau \delta - \delta + \pi = 0$$
$$f'(K) - \delta = r - \frac{\pi}{1 - \tau_c}$$

Partial and general equilibrium analysis

Firms choose borrowing B to maximize value taking into account inflation.

$$max_B \quad V = (f(K) - rB)(1 - \tau_c) + \tau_c \delta K - \delta K + \pi B$$

Take the first-order condition.

$$\partial V / \partial B = (1 - \tau_c) f'(K) - (1 - \tau_c) r + \tau \delta - \delta + \pi = 0$$
$$f'(K) - \delta = r - \frac{\pi}{1 - \tau_c}$$

- Need to consider how r changes with π, (general equilibrium).

General equilibrium analysis

To think general equilibrium, we need to allow multiple variables to change at the same time.

First-order condition

$$f'(K) - \delta = r - \frac{\pi}{1 - \tau_c}$$

What variables do we think change?

1. Let capital change K.
2. Let interest rates change r.
3. Let inflation change π

Total differentiation allowing k, π, r to change

Totally differentiate the first-order condition. Note $f''(K) < 0$.

$$f'(K) - \delta = r - \frac{\pi}{1-\tau_c}$$

$$\text{totally differentiate} \quad f''(K)dK = dr - \frac{d\pi}{1-\tau_c}$$

$$\frac{dK}{d\pi} = -\frac{1}{-f''(K)}\left(\frac{dr}{d\pi} - \frac{1}{1-\tau_c}\right)$$

How capital adjusts depends on how interest rates change.

- How capital responds to inflation depends on how much interest rates respond to inflation.

$$\frac{dK}{d\pi} = -\frac{1}{-f''(K)}\left(\frac{dr}{d\pi} - \frac{1}{1-\tau_c}\right)$$

$$\frac{dK}{d\pi} = \begin{cases} > 0, & \text{if } \frac{dr}{d\pi} < \frac{1}{1-\tau_c} \\ = 0, & \text{if } \frac{dr}{d\pi} = \frac{1}{1-\tau_c} \\ < 0, & \text{if } \frac{dr}{d\pi} > \frac{1}{1-\tau_c} \end{cases}$$

How do interest rates change with inflation?

1. To know whether investment increases or decreases with inflation, we need to know how interest rates change with inflation.
2. Remember, Fisher 1930 noted $dr/d\pi = 1$.
3. To solve it in general equilibrium, we need to consider supply of capital (lenders).
4. Lenders receive real after-tax returns (individual tax rate t):

$$\tilde{r} = r(1 - t) - \pi$$

Totally differentiate this allowing $\tilde{r}$, r, and π to change.

Supply of capital comparative statics

Lenders receive real after-tax returns (individual tax rate t):

$$\tilde{r} = r(1 - t) - \pi$$

Totally differentiate

$$d\tilde{r} = (1 - t)dr - d\pi$$

$$\frac{d\tilde{r}}{d\pi} = (1 - t)\frac{dr}{d\pi} - 1$$

Change in market supply of capital

$$\frac{d\tilde{r}}{d\pi} = (1-t)\frac{dr}{d\pi} - 1$$

Market supply of capital

$$\frac{d\tilde{r}}{d\pi} = \begin{cases} < 0, & \text{if } \frac{dr}{d\pi} < \frac{1}{1-t} \\ = 0, & \text{if } \frac{dr}{d\pi} = \frac{1}{1-t} \\ > 0, & \text{if } \frac{dr}{d\pi} > \frac{1}{1-t} \end{cases}$$

For capital markets to clear supply = demand

Change in market supply $\frac{d\tilde{r}}{d\pi}$

Change in market demand $\frac{dK}{d\pi}$

If $\frac{dr}{d\pi} < \frac{1}{1-t}$ then $\frac{d\tilde{r}}{d\pi} < 0$ and $\frac{dK}{d\pi} > 0$. Nope

If $\frac{dr}{d\pi} > \frac{1}{1-t}$ then $\frac{d\tilde{r}}{d\pi} > 0$ and $\frac{dK}{d\pi} < 0$. Nope

If $\frac{dr}{d\pi} = \frac{1}{1-t}$ then $\frac{d\tilde{r}}{d\pi} = 0$ and $\frac{dK}{d\pi} = 0$. Yup

Implications

1. Interest rate increases more than inflation $\frac{dr}{d\pi} = \frac{1}{1-\tau_c}$.

2. Interest rate adjusts for inflation **AND** tax implications.

3. Capital is unaffected by inflation $\frac{dK}{d\pi} = 0$.

Partial versus general equilibrium

Empirical evidence is often partial equilibrium.

As we saw, implications can be very different in general equilibrium.

In general equilibrium—anything is possible.

B.2 The envelope theorem

How do corporate tax rates affect total value in the economy?

Total value (welfare) in the economy

- So far, we have considered firm value solely.
- For tax policy, we may want to consider additional effects of corporate taxes.
- What do we need to include in the model to capture total value in the economy?
- How do corporate taxes distort welfare?

How do corporate taxes distort welfare?

There are several candidates

1. Change firm behavior due to changes in capital K.
2. Change taxable income $Y(K, \rho)$ and thus tax revenues.
3. Change tax reporting ρ of firms.
   - Let fraction μ of firm reporting be a shift in value and $1 - \mu$ be a resource cost.
   - Examples of shifting are transfers to accounting firms or shifting money into a tax preferred vehicle.
   - Examples of resource costs include exerting effort in a law library figuring out credits and deductions.
   - Does it matter if it is a resource cost or shifting?

A marginal increase in the corporate tax rate distorts total value in the economy by

- Taking money away from firms
- Decreasing firm value
- Changing tax revenue
- Increasing costs to avoid taxes

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Firms maximize firm value

Write firm value in second period value

$$max_{K,\rho} \quad V = -rK + (1 - \tau_c)(f(K) - \rho) + \rho - c(\rho)$$

- Firms choose capital K and amount of reporting ρ.
- Taxable income $Y = f(K) - \rho$.
- Cost of reporting $c(\rho)$ and benefit of reporting $\tau_c \rho$.
- Profits $f(K)$.

Total value in the economy

Total value in the economy.

$$\begin{aligned} TV = & [-rK + (1-\tau_c)(f(K)-\rho) + \rho - c(\rho)] & \text{Firm value} \\ & + \tau_c(f(K)-\rho) & \text{Tax revenue} \\ & + \mu c(\rho) & \text{Cost of reporting} \end{aligned}$$

Cost of reporting to the extent that it shifts to accounting and law firms and is not a resource cost.

- Pure shift of value $\mu = 1$.
- Pure resource cost $\mu = 0$.

How does total value in the economy change with taxes?

Total value in the economy, where $Y(k,\rho) = f(K) - \rho$

$$
\begin{aligned}
TV = & [-rK + (1-\tau_c)Y(k,\rho) + \rho - c(\rho)] && \text{Firm value} \\
& + \tau_c Y(k,\rho) && \text{Tax revenue} \\
& + \mu c(\rho) && \text{Cost of reporting}
\end{aligned}
$$

Take the derivative with respect to $(1-\tau_c)$, where k and ρ are functions of $(1-\tau_c)$.

Derivative in pieces

$$TV = [-rK + (1-\tau_c)Y(K,\rho) + \rho - c(\rho)] + \tau_c Y(K,\rho) + \mu c(\rho)$$

Piece 1: direct effect

$$\frac{\partial TV}{\partial(1-\tau_c)} = Y(K,\rho) - Y(K,\rho) \qquad \text{direct effect} = 0$$

The direct effect is a transfer from firms to the government

Derivative in pieces

$$TV = [-rK + (1-\tau_c)Y(K,\rho) + \rho - c(\rho)] + \tau_c Y(K,\rho) + \mu c(\rho)$$

Piece 2: tax revenue and cost of reporting

$$\frac{\partial TV}{\partial(1-\tau_c)} =$$

$$\tau_c \frac{\partial Y(K,\rho)}{\partial K}\frac{\partial K}{\partial(1-\tau_c)} + \tau_c \frac{\partial Y(K,\rho)}{\partial \rho}\frac{\partial \rho}{\partial(1-\tau_c)} + \mu c'(\rho)\frac{\partial \rho}{\partial(1-\tau_c)}$$ tax revenue and cost of reporting

Piece 3 firm value indirect effect (through k and ρ)

$$TV = [-rK + (1-\tau_c)Y(K,\rho) + \rho - c(\rho)] + \tau_c Y(K,\rho) + \mu c(\rho)$$

$$\frac{\partial V}{\partial(1-\tau_c)} = \underbrace{\left(-r + (1-\tau_c)\frac{Y(K,\rho)}{\partial K}\right)}_{=0 \text{ bc FOC}} \frac{\partial K}{\partial(1-\tau_c)}$$

$$+ \underbrace{\left((1-\tau_c)\frac{Y(K,\rho)}{\partial \rho} + 1 - c'(\rho)\right)}_{=0 \text{ bc FOC}} \frac{\partial \rho}{\partial(1-\tau_c)} = 0$$

This is the envelope theorem

The derivative is only piece 2

$$TV = [-rK + (1-\tau_c)Y(K,\rho) + \rho - c(\rho)] + \tau_c Y(K,\rho) + \mu c(\rho)$$

$$\frac{\partial TV}{\partial(1-\tau_c)} = \tau_c \frac{\partial Y(K,\rho)}{\partial K}\frac{\partial K}{\partial(1-\tau_c)} + \tau_c \frac{\partial Y(K,\rho)}{\partial \rho}\frac{\partial \rho}{\partial(1-\tau_c)} + \mu c'(\rho)\frac{\partial \rho}{\partial(1-\tau_c)}$$

How does total value in the economy change with tax rates?

1. Taking money from firms?
   - No, the direct effect is zero—transfer from firms to the government.
2. Firm value?
   - No, the indirect effect of firm value is zero by the envelope theorem.
3. Tax revenue changes?
   - Yes.
4. Tax reporting?
   - Yes, if reporting is shifting $\mu > 0$.

What are other examples of the envelope theorem?

1. Shepard's lemma: in a cost minimization problem the derivative with respect to the interest rate is capital and the derivative with respect to wages is labor.
2. Le Chatelier's principle: labor is more responsive to a change in the wage in the long run than in the short run because in the long run the firm can adjust its capital.
3. Deadweight loss Harberger (1964) "triangle."

B.3 Sufficient statistics

How do corporate tax rates affect total value in the economy?

How familiar are you with the concept of sufficient statistic?

- Very familiar, all of my research is based on sufficient statistics
- Familiar
- Not that familiar
- I have heard the words, but do not know what they mean, and am too a…
- I have blissfully never heard these words before

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Is there one parameter that can tell us about distortions in the economy?

1. Feldstein (1999) argued that the elasticity of taxable income with respect to the corporate tax rate captured the welfare gain/cost from taxes.
   - The elasticity of taxable income as a sufficient statistic for welfare analysis.
   - For more on sufficient statistics see Chetty (2009).
2. Many papers have qualified this statement (Doerrenberg, Peich, and Siegloch, 2017; Coles, Patel, Seegert, and Smith, 2021).
3. Follow the analysis in Coles, Patel, Seegert, and Smith (2021) to
   - Demonstrate sufficient statistics.

Total value in the economy

Start again with total value in the economy.

$$\begin{aligned} TV = & [-rK + (1-\tau_c)Y(K,\rho) + \rho - c(\rho)] && \text{Firm value} \\ & + \tau_c Y(K,\rho) && \text{Tax revenue} \\ & + \mu c(\rho) && \text{Cost of reporting} \end{aligned}$$

Is there one parameter that would be sufficient for understanding $\partial TV/\partial(1-\tau_c)$?

Derive welfare costs of corporate taxes

Take the derivative of total value with respect to the net-of-tax rate.

$$\frac{\partial TV}{\partial(1-\tau_c)} = \tau_c \frac{\partial Y(K,\rho)}{\partial K}\frac{\partial K}{\partial(1-\tau_c)} + \tau_c \frac{\partial Y(K,\rho)}{\partial \rho}\frac{\partial \rho}{\partial(1-\tau_c)} + \mu c'(\rho)\frac{\partial \rho}{\partial(1-\tau_c)}$$

$$= \tau_c \frac{\partial Y(K,\rho)}{\partial(1-\tau_c)} + \mu c'(\rho)\frac{\partial \rho}{\partial(1-\tau_c)}$$

Derive welfare costs of corporate taxes

Take the derivative of total value with respect to the net-of-tax rate.

$$\frac{\partial TV}{\partial(1-\tau_c)} = \tau_c \frac{\partial Y(K,\rho)}{\partial(1-\tau_c)} + \mu c'(\rho) \frac{\partial \rho}{\partial(1-\tau_c)}$$

Rearrange to get terms that we like (note $c'(\rho) = \tau_c$).

$$\frac{\partial TV}{\partial(1-\tau_c)} = \frac{\tau_c}{1-\tau_c} Y \frac{\partial Y(K,\rho)}{\partial(1-\tau_c)} \frac{1-\tau_c}{Y} + \mu \frac{\tau_c}{1-\tau_c} \rho \frac{\partial \rho}{\partial(1-\tau_c)} \frac{1-\tau_c}{\rho}$$

Derive welfare costs of corporate taxes

Rearrange to get terms that we like (note $c'(\rho) = \tau_c$).

$$\frac{\partial TV}{\partial(1-\tau_c)} = \frac{\tau_c}{1-\tau_c} Y \frac{\partial Y(K,\rho)}{\partial(1-\tau_c)} \frac{1-\tau_c}{Y} + \mu \frac{\tau_c}{1-\tau_c} \rho \frac{\partial \rho}{\partial(1-\tau_c)} \frac{1-\tau_c}{\rho}$$

Rewrite in terms of elasticities

$$\frac{\partial TV}{\partial(1-\tau_c)} = Y \frac{\tau_c}{1-\tau_c} e_Y - \mu\rho \frac{\tau_c}{1-\tau_c} e_\rho$$

Is the elasticity of taxable income a sufficient statistic?

We know that

$$\frac{\partial TV}{\partial(1-\tau_c)} = Y\frac{\tau_c}{1-\tau_c}e_Y - \mu\rho\frac{\tau_c}{1-\tau_c}e_\rho$$

1. If the cost of tax adjustments is a resource cost ($\mu = 0$), then
   - the elasticity of taxable income is a sufficient statistic for the distortion to total value.
2. If the cost of tax adjustments is partially a transfer ($\mu > 0$), then
   - the elasticity of taxable income is an upper bound on the distortion to total value
   - the distortion to total value decreases with the tax adjustment elasticity e_τ

B.4 Structural parameter estimation

How elastic are firms?

Structural estimation

Structural estimation connects the model directly to the empirical estimation.

1. This can be as simple as running an OLS regression.
2. Alternatively, it could require estimation via general method of moments, maximum likelihood, or simulated method of moments.
3. What are the benefits?
    - Identifies exactly what your empirical estimation is telling you.
    - Allows for extrapolation out of sample for policy “experiments."

Would you consider using a structural model in your research?

- Yes, definitely
- Yes, but I do not know how
- No, I do not know how
- No, I do not see the point

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Standard model of firms with fixed cost

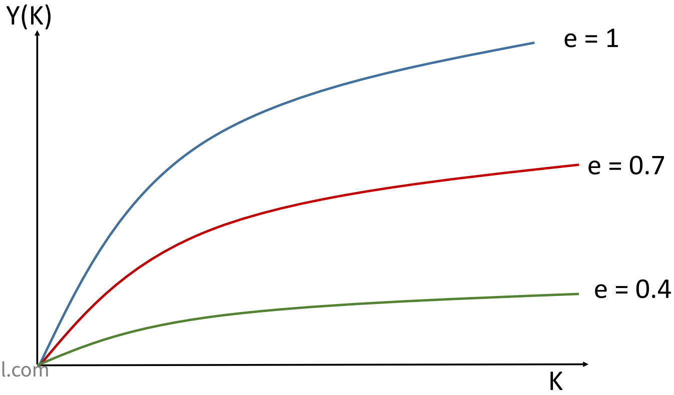
- Firm i chooses how much earnings to distribute as a dividend ($D_i \geq 0$) and how much equity to issue ($E_i \geq 0$).
- Those choices determine period 2 capital: $K_{2,i} = K_{1,i} + E_i - D_i$.
- Profits net depreciation costs:

$$Y_i(K_{2,i}) = \frac{1+e}{e} A_i^{\frac{1}{1+e}} K_{2,i}^{\frac{e}{1+e}} - F_i.$$

- Fixed costs $F_i = exp(X_F' \beta_F + \nu_F)$, normally distributed.
- Productivity $A_i = exp(X_A' \beta_A + \nu_A)$, normally distributed.
- Parameter of interest e tells us how elastic firms are.

Parameter e tells us how elastic firms are

$$Y_i(K_{2,i}) = \frac{1+e}{e} A_i^{\frac{1}{1+e}} K_{2,i}^{\frac{e}{1+e}} - F_i.$$



Firm optimization with linear tax schedule

$$max_{K_{2,i}} \quad V = -K_{2,i} + \frac{(1-\tau)Y_i(K_{2,i})}{1+r}$$

First-order condition

$$\frac{\partial V}{\partial K} = -1 + \frac{1}{1+r}(1-\tau)Y_i'(K_{2,i}) = 0$$

$$Y_i'(K_{2,i}) = \frac{1+r}{1-\tau}$$

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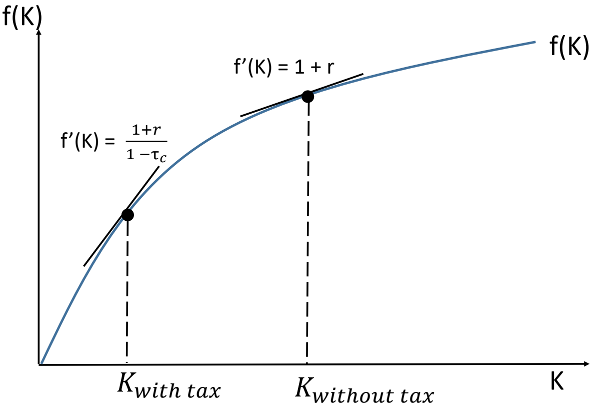
Distortions to investment from corporate taxes and equity



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Guts

$$Y_i(K_{2,i}) = \frac{1+e}{e} A_i^{\frac{1}{1+e}} K_{2,i}^{\frac{e}{1+e}} - F_i.$$

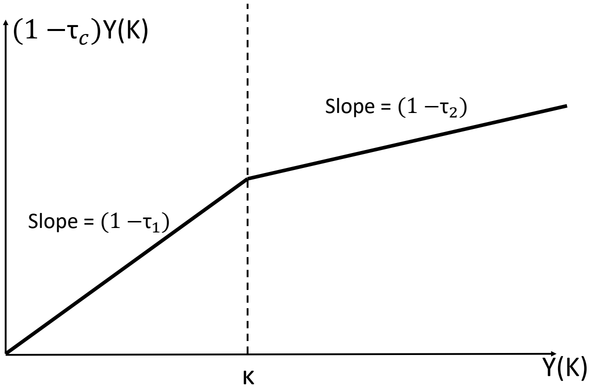
$$Y_i'(K_{2,i}) = A_i^{\frac{1}{1+e}} K_{2,i}^{\frac{-1}{1+e}} = \frac{1+r}{1-\tau}$$

$$K_{2,i} = A_i \left(\frac{1-\tau}{1+r} \right)^{1+e}$$

$$Y_i(K_{2,i}) = \frac{1+e}{e} A_i (1+r)^{-e} (1-\tau)^e - F_i.$$

Now consider a tax schedule with a kink in it

Profits below κ taxed at rate τ_1 and profits above κ taxed at rate τ_2, where $\tau_1 < \tau_2$.



Firms maximize value subject to the corporate tax schedule

Profits below κ taxed at rate τ_0 and profits above κ taxed at rate τ_1, where $\tau_1 < \tau_2$.

$$max_{K_{2,i}} \quad V = -K_{2,i} + \mathbb{1}(Y_i(K_{2,i}) \leq \kappa)\frac{(1-\tau_0)Y_i(K_{2,i})}{1+r} + \mathbb{1}(Y_i(K_{2,i}) > \kappa)\frac{(1-\tau_0)\kappa + (1-\tau_1)(Y_i(K_{2,i}) - \kappa)}{1+r}$$

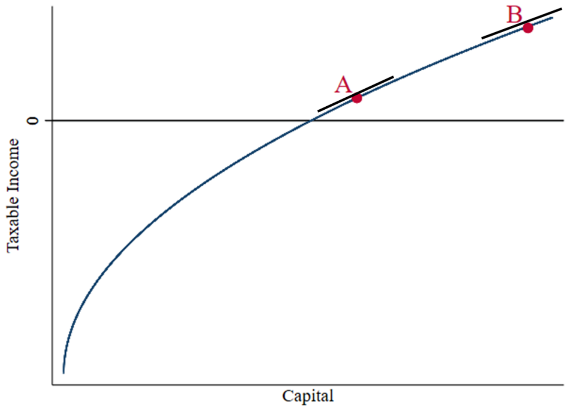
Let $\kappa = 0$, such that τ_0 for $Y < 0$ and τ_1 for $Y > 0$

Three cases

1. Firms with $Y > 0$ with higher tax rate τ_1

$$Y_i'(K_{2,i}) = \frac{1 + r}{1 - \tau_1}$$

This firm reports taxable income at point A above the kink



- Point A has a slope $(1 + r)/(1 - \tau_1)$.
- Point B has a slope $(1 + r)/(1 - \tau_0)$.

Three cases

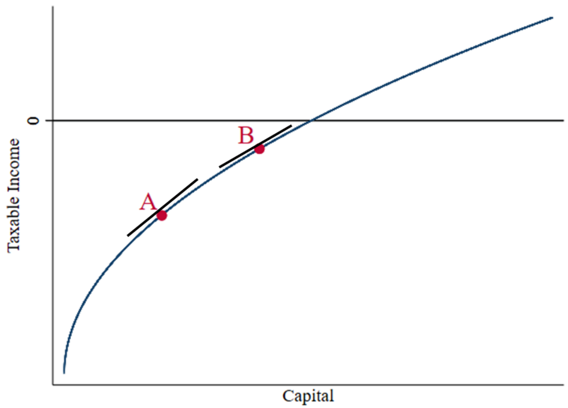
1. Firms with $Y > 0$ with higher tax rate τ_1

$$Y_i'(K_{2,i}) = \frac{1+r}{1-\tau_1}$$

2. Firms with $Y < 0$ with the lower tax rate τ_0

$$Y_i'(K_{2,i}) = \frac{1+r}{1-\tau_0}$$

This firm reports taxable income at point B below the kink



- Point A has a slope $(1 + r)/(1 - \tau_1)$.
- Point B has a slope $(1 + r)/(1 - \tau_0)$.

Three cases

1. Firms with $Y > 0$ with higher tax rate τ_1

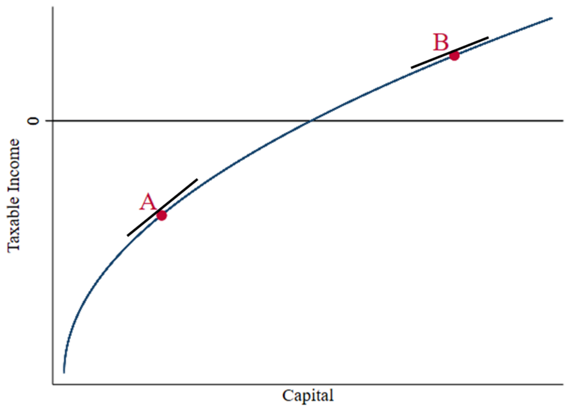
$$Y_i'(K_{2,i}) = \frac{1+r}{1-\tau_1}$$

2. Firms with $Y < 0$ with the lower tax rate τ_0

$$Y_i'(K_{2,i}) = \frac{1+r}{1-\tau_0}$$

3. Firms with $Y > 0$ with low tax rate and $Y < 0$ with high tax rate.

This firm reports taxable income at the kink



- Point A has a slope $r/(1 - t_1)$.
- Point B has a slope $r/(1 - t_0)$.

In this case, the firm bunches at the kink

Three cases with kink at zero

1. Firms with $Y > 0$ with higher tax rate τ_1

$$Y_i'(K_{2,i}) = \frac{1 + r}{1 - \tau_1}$$

2. Firms with $Y < 0$ with the lower tax rate τ_0

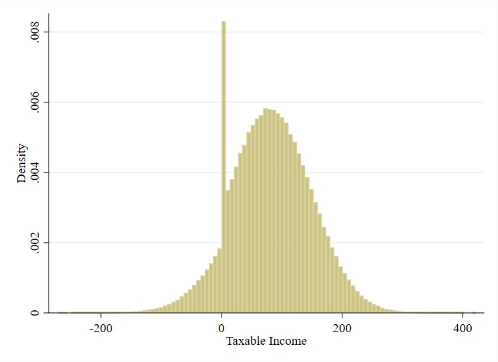
$$Y_i'(K_{2,i}) = \frac{1 + r}{1 - \tau_0}$$

3. Firms with $Y > 0$ with low tax rate and $Y < 0$ with high tax rate.

$$Y_i(K_{2,i}) = 0$$

Taxable income is given by piecewise function

$$Y_i^* = \begin{cases} \frac{1+e}{e} r^{-e}(1-\tau_0)^e A_i - F_i, & A_i \leq \underline{A}(e,\kappa,\tau_1) \\ \kappa, \quad \underline{A}(e,\kappa,\tau_0) < A_i < & \bar{A}(e,\kappa,\tau_1) \\ \frac{1+e}{e} r^{-e}(1-\tau_1)^e A_i - F_i, & A_i \geq \bar{A}(e,\kappa,\tau_1) \end{cases}$$



Substitutions to get to estimation equation

$$Y_i = \frac{1+e}{e} r^{-e}(1-\tau_j)^e A_i - F_i = \lambda_j A_i - F_i$$

- Fixed costs $F_i = \beta_F X_F + \nu_F$
- Productivity $A_i = \beta_A X_A + \nu_A$

$$\begin{aligned} Y &= \lambda_j(\beta_A X_A + \nu_A) - \beta_F X_F - \nu_F \\ &= \lambda_j \beta_A X_A - \beta_F X_F + \lambda_j \nu_A - \nu_F \\ &= \delta_{A,j} X_A + \delta_F X_F + \tilde{\nu}_j \end{aligned}$$

Starting to look like something I could estimate

Using variation created by the kink

Above the kink

$$Y = \delta_{A,1} X_A + \delta_F X_F + \tilde{\nu}_1$$

Below the kink

$$Y = \delta_{A,0} X_A + \delta_F X_F + \tilde{\nu}_0$$

Need to use Heckman correction for the selection above and below kink.

$$\mathbb{E}[Y | X_A, X_F, Y < \kappa] = \delta_{A,0} X_A + \delta_F X_F - w_1 \frac{\phi\left(\frac{\delta_{A,0} X_A + \delta_F X_F}{w_1}\right)}{\Phi\left(\frac{\delta_{A,0} X_A + \delta_F X_F}{w_1}\right)}$$

Now with parameter estimates

1. Ratio of coefficients on productivity:

$$\frac{\delta_{A,0}}{\delta_{A,1}} = \frac{\beta_A^{\frac{1+e}{e}}(1+r)^{-e}(1-\tau_0)^e}{\beta_A^{\frac{1+e}{e}}(1+r)^{-e}(1-\tau_2)^e} = \frac{(1-\tau_0)^e}{(1-\tau_1)^e}$$

2. Derive the parameter e

$$e = ln\left(\frac{\delta_{A,0}}{\delta_{A,1}}\right)\frac{1}{ln(1-\tau_0) - ln(1-\tau_1)}$$

Stata code to do this provided below

Nathan Seegert — Home Vitae Research Teaching Media Code

The Elasticity of Taxable Income Across Countries

with Claudio Agostini, Marinho Bertanha, Govindadeva Bernier, Katarzyna Bilicka, Jaroslav Bukovina, Yuxuan He, Evangelos Koumanakos, Tomas Lichard, Jan Palguta, Elena Patel, Louis Perrault, Kristina Strohmaier, Maximilian Todtenhaupt, and Branislav Zudel.

This package is in *beta testing*. This package is not yet a complete package (ado file program) and is currently being tested with real data. Feel free to test the code or build off of it for your own work. As a courtesy, please let me know if you find any errors.

STATA - manual installation

1. download and unzip the file **"stata_crosscountry.zip"**
2. This folder contains four files.
   a) 0_simulateData.do This file lets you simulate data to be used with the other files.
   b) 1_fixedCost_2021129.do This file implements the fixed cost method described in the paper.
   c) 2_TwoStep_20220201.do This file implements the two step method described in the paper.
   d) twostep_bserrors.ado This file is used to produce standard errors.

- www.NathanSeegert.com/code

Conclusion

Models focus the reader on the tradeoff in your work

Modeling takeaways:

1. Begin with the tradeoff you are interested in studying.
   - Define the players, strategies, and payoffs.
2. Add in features of interest.
   - e.g., Depreciation schedules, Tax reporting, etc.
3. Let your model ebb and flow.
   - Add features to test whether conclusions are robust.
   - Delete features that are robust.
4. Have fun and be creative.

Thank you for some fun modeling

BONUS: User cost of capital and effective tax rate ETR

How do tax depreciation methods distort investment?

Tax rules on investment

We want to understand how tax rules impact investment.

1. Firms have depreciation allowance a_t at time t on a dollar of investment.
   - Accelerated depreciation or any other schedule.
   - $\int a_t dt = 1$, and $z \equiv \int e^{-\rho t} a_t dt$.
   - Capital depreciates exponentially at rate δ; $K_t = Ee^{-\delta t}$.
   - Firms may receive a contemporaneous investment tax credit of κ per dollar invested.
2. We could do this in continuous time (and most of the literature does), but we can get a lot from just a two period model.
3. Modeling goals: explore how to use the user cost of capital and effective tax rate ETR to investigate tax distortions.

Investment with depreciation and discount rate

To follow the continuous time literature, we can update the model as below:

$$max_E \quad V = D - cE + \frac{(1-\tau_c)f(K)}{\delta + \rho} + \tau_c zE + \kappa E$$

1. E is equity.
2. c is after-tax cost of putting a dollar into the firm.
3. $K = X - D + E$ is capital in period 2.
4. δ is the capital depreciation rate.
5. ρ is the rate at which owners discount after-tax flows.
6. z is the depreciation allowance.
7. κ is the investment tax credit.

Tax adjusted user cost of capital

First order condition

$$\partial E : - c + \frac{(1 - \tau_c) f'(K)}{\delta + \rho} + \tau_c z + \kappa = 0$$

$$f'(K) = \frac{c - \kappa - \tau_c z}{1 - \tau_c} (\rho + \delta)$$

- The right side is the user cost of capital.

Tax adjusted user cost of capital

User cost of capital

$$f'(K) = \frac{c - \kappa - \tau_c z}{1 - \tau_c}(\rho + \delta)$$

1. If $c = 1$, then this is the Hall-Jorgenson tax-adjusted user cost of capital.
2. If $\tau_c = 0$, the rental cost of capital is $c(\rho + \delta)$, which reflects the time value of money and cost of depreciation interacted with the expenditure level.
3. Everything else, is the impact of taxation.

Consider different depreciation methods

1. Let investments be depreciated at economic depreciation, then $z = \delta/(\rho + \delta)$.
2. Let investments be expensed immediately, then $z = 1$.
   - If $\kappa = 0$ and $c = 1$, then we can see that immediate expensing returns us to the cost of capital without taxes.

$$
\begin{aligned}
f'(K) &= \frac{c - \kappa - \tau_c z}{1 - \tau_c}(\rho + \delta) \\
&= \frac{1 - \tau_c}{1 - \tau_c}(\rho + \delta) \\
&= \rho + \delta
\end{aligned}
$$

- Obviously, depreciation is more complicated than either of these scenarios in practice.

Effective tax rates (ETR)

Consider the investment level induced by the condition:

$$\rho \equiv [f'(K) - \delta](1 - ETR).$$

that defines the effective tax rate

$$ETR = \frac{f'(K) - \delta - \rho}{f'(K) - \delta}.$$

The ETR provides the “single" tax rate that produces the same investment level given by a combination of tax parameters.

Combinations of tax parameters

User cost of capital

$$f'(K) = \frac{c - \kappa - \tau_c z}{1 - \tau_c}(\rho + \delta)$$

Now, we can consider different combinations of tax parameters and find the effective tax rate.

- Economic deprecation $z = \delta/(\rho + \delta)$.
- Immediate expensing $z = 1$.
- Equity financed investment $c = 1$.
- Debt financed investment $c < 1$.
- investment tax credit κ.

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Effective tax rates (ETR) scenario 1

General model

$$max_E \quad V = D - cE + \frac{(1-\tau_c)f(K)}{\delta + \rho} + \tau_c zE + \kappa E$$

General user cost of capital:

$$f'(K) = \frac{c - \kappa - \tau_c z}{1 - \tau_c}(\rho + \delta)$$

Scenario 1: Consider a firm with

1. Equity-financed investment $c = 1$.
2. No investment tax credit $\kappa = 0$
3. Immediate expensing of investment $z = 1$.

Effective tax rates (ETR) scenario 1

General user cost of capital:

$$f'(K) = \frac{c - \kappa - \tau_c z}{1 - \tau_c}(\rho + \delta)$$

Scenario 1: Consider a firm with

1. Equity-financed investment $c = 1$.
2. No investment tax credit $\kappa = 0$
3. Immediate expensing of investment $z = 1$.

Scenario 1 user cost of capital:

$$f'(K) = \rho + \delta$$

Effective tax rates (ETR) scenario 1

Scenario 1 user cost of capital $c = 1$, $\kappa = 0$, and $z = 1$:

$$f'(K) = \rho + \delta$$

Substituting this into our ETR, we get

$$ETR = \frac{f'(K) - \delta - \rho}{f'(K) - \delta} = \frac{\rho + \delta - \delta - \rho}{\rho + \delta - \delta} = 0$$

- In this scenario immediate expensing leads to no distortions!

Effective tax rates (ETR) scenario 2

General model

$$max_E \quad V = D - cE + \frac{(1-\tau_c)f(K)}{\delta + \rho} + \tau_c zE + \kappa E$$

General user cost of capital:

$$f'(K) = \frac{c - \kappa - \tau_c z}{1 - \tau_c}(\rho + \delta)$$

Scenario 2: Consider a firm with

1. Equity-financed investment $c = 1$.
2. No investment tax credit $\kappa = 0$
3. Depreciation allowances equal to economic depreciation $z = \delta/(\rho + \delta)$.

Effective tax rates (ETR) scenario 2

General user cost of capital:

$$f'(K) = \frac{c - \kappa - \tau_c z}{1 - \tau_c}(\rho + \delta)$$

Scenario 2: Consider a firm with

1. Equity-financed investment $c = 1$.
2. No investment tax credit $\kappa = 0$
3. Depreciation allowances equal to economic depreciation $z = \delta/(\rho + \delta)$.

Scenario 2 user cost of capital:

$$f'(K) = \rho/(1 - \tau_c) + \delta$$

Effective tax rates (ETR) scenario 2

Scenario 2 user cost of capital $c = 1, \kappa = 0, z = \delta/(\rho + \delta)$:

$$f'(K) = \rho/(1 - \tau_c) + \delta$$

Substituting this into our ETR, we get

$$ETR = \frac{f'(K) - \delta - \rho}{f'(K) - \delta} = \frac{\rho/(1 - \tau_c) + \delta - \delta - \rho}{\rho/(1 - \tau_c) + \delta - \delta} = \tau_c$$

- In this scenario economic depreciation leads to a distortion that increases with the corporate tax rate.

Effective tax rates (ETR) scenario 3

General model

$$max_E \qquad V = D - cE + \frac{(1-\tau_c)f(K)}{\delta + \rho} + \tau_c zE + \kappa E$$

General user cost of capital:

$$f'(K) = \frac{c - \kappa - \tau_c z}{1-\tau_c}(\rho + \delta)$$

Scenario 3: Consider a firm with

1. Debt-financed investment $c = 1 - \tau_c$.
   - $c = (r(1-\tau_c) + \delta)/(\rho + \delta) = 1 - \tau_c$
   - $c = 1 - \tau_c$ with the simplification, $\delta = 0$, $\rho = r$.
2. No investment tax credit $\kappa = 0$
3. depreciation allowances equal to economic depreciation $z = \delta/(\rho + \delta)$.

Effective tax rates (ETR) scenario 3

General user cost of capital:

$$f'(K) = \frac{c - \kappa - \tau_c z}{1 - \tau_c}(\rho + \delta)$$

Scenario 3: Consider a firm with

1. Debt-financed investment $c = 1 - \tau_c$.
   - $c = (r(1 - \tau_c) + \delta)/(\rho + \delta) = 1 - \tau_c$
   - $c = 1 - \tau_c$ with the simplification, $\delta = 0$, $\rho = r$.
2. No investment tax credit $\kappa = 0$
3. depreciation allowances equal to economic depreciation $z = \delta/(\rho + \delta)$.

Scenario 3 user cost of capital:

$$f'(K) = \rho$$

Effective tax rates (ETR) scenario 3

Scenario 3 user cost of capital $c = 1 - \tau_c$, $\delta = 0$, $\kappa = 0$ and $z = \delta/(\rho + \delta)$:

$$f'(K) = \rho$$

Substituting this into our ETR, we get

$$ETR = \frac{f'(K) - \delta - \rho}{f'(K) - \delta} = \frac{\rho - \rho}{\rho} = 0$$

- If there is debt finance and tax depreciation is economic deprecation there is no distortion.
- If there is debt finance and tax depreciation that is more rapid than economic depreciation, then the ETR is **negative**.

ETR can be used to measure/investigate distortions

ETR = 0 implies no distortion from taxation. This occurs when

1. Equity financing of investment and immediate expensing.
2. Debt financing of investment and depreciation is allowed at economic depreciation.
    - In both cases, all investment costs are deductible.

ETR and user cost of capital are helpful to understand when and how taxes distort investment.

What else might be important in this model?

1. What other depreciation schedules might we want to model and how would they change investment behavior?

2. What other behavior may depreciation schedules change?

Empirical evidence

Use changes in depreciation (via bonus depreciation) to look at effect on investment.

1. Early literature found large investment responses (House and Shapiro, 2008; Zwick and Mahon, 2017).
   - Use differences across industries in investment.
   - Manufacturing longer lived capital than software developers and thus have more benefits from bonus depreciation.
2. These estimates might be too large though if competition is not taken into account (Patel and Seegert, 2020).
   - Investment is a strategic variable and responses to tax incentives depend on how competitive or concentrated the market is.
   - Industries with longer lived capital likely also more concentrated due to large fixed costs.

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